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Research Article

Prediction of Life Cycle of Induction Furnace Wall for Silica Ramming Mass

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Abstract

Casting process involves melting for metals in closed vessel called Furnace. Induction furnaces are using principle of electromagnetics so any kind of fuel is not required. The problem is low cycle thermal fatigue failure of induction melting furnace wall. It is having micro cracks and it loses its thermal and mechanical properties. It creates problematic situation to ongoing production cycle. Life cycle varies depending upon size and geometry of induction furnace, change in application, material which is to be melted, worker's ability and skills, scrap material used for melting, etc. It is only possible to predict life cycle of induction furnace wall by various experimental work for different cases. There is no standard or reliable theory which can predict life cycle of induction furnace wall till date. In this paper, an innovative algorithm is developed which can predict the life cycle of induction furnace wall with very high accuracy. The induction melting furnace wall is made up of ceramic based silica ramming mass. The failure of furnace wall occurs due to creation of reversible cyclic thermal stresses by heating and cooling cycles. Temperature field and thermal stress field of the induction furnace wall are calculated by using explicit finite difference method based on the physical explanation of its damage under low cycle thermal fatigue conditions. The life time of the furnace wall is found out by means of critical thermal stresses created inside the wall of induction furnace from modified S – log N Curve.

Keywords: *Life cycle prediction, Induction furnace, explicit finite difference analysis, refractory, silica ramming mass*

1. Introduction

Casting is the oldest manufacturing technology as it was useful for making arrow heads of copper dates back to 4000 B.C. Casting process basically involves melting of metals and pouring it into the cavity so that after solidification process, it will take the shape required. Starting from simple to very complicated shapes can also be manufactured using melting of metals in casting process. Castings can be of variable sizes starting from few mm long artificial teeth to 12-meter-long ship propeller.

With the help of casting process very complex parts are also possible to be manufactured with a very high accuracy. Casting process can create both external as well as internal shapes of components. Casting produces isotropic materials which are having same mechanical and physical properties along any direction. Casting is a very economical manufacturing process and even waste material is also re melted and reused again. Closed vessel used in casting for melting of metals and non-metals is called Furnace. (John Campbell, Castings)

Induction furnaces are using electricity for melting so fuel is not required. Induction melting furnaces are mostly used in the cast iron industry. Ceramic based furnace wall of induction furnace is an important component which is used as insulation layer. It is made of monolithic refractories ramming materials like silica, alumina, magnesia etc. The rammed wall is directly under the effect of the thermal cycling of the high temperature molten cast iron in the furnace. Thermal mechanical fatigue damage is easy to happen for it because of the larger phase transformation thermal stresses so it has a shorter life span. It creates a problem to ongoing production cycle. It also takes a lot of time to replace the ceramic based refractory wall. (A V K Suryanarayana, Fuels Furnaces Refractory and Pyrometry).

The research on the distribution rule of temperature and thermal stress field and on the fatigue life assessment method for the refractory wall will not only lay foundation for the study on the thermal fatigue of this kind of parts under thermal shock condition of low cycle and high phase transition stresses but also offers effective control for thermal fatigue failure.

A comprehensive literature review on computational investigation on different kinds of

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furnaces is done to study research trends. (Nirajkumar Mehta, May 2012). A review is done on applications of different numerical methods in heat transfer with its applications. (N C Mehta, Vipul B Gondaliya *et al*, February 2013). Thermal fatigue analysis of induction melting furnace wall is done for silica ramming mass. (N C Mehta, Akash D Raiyani *et al*, February 2013). A review is done for research on induction heating. (Vimal R Nakum *et al*, April 2013). A review is done for metal forming analysis using different numerical methods. (N C Mehta *et al*, May 2013). Transient heat transfer analysis of induction furnace is done by using finite element analysis. (VipulGondaliya *et al*, August 2013). Thermal fatigue analysis of induction furnace wall is done for alumina ramming mass. (N C Mehta *et al*, October 2013).

Thermal analysis of hot wall condenser is done for domestic refrigerator using numerical method for temperature distribution. (Akash D Raiyani *et al*, July 2014). Optimisation of wall thickness is done for minimum heat loss for induction furnace by finite element analysis. (Dipesh D Shukla *et al*, December 2014). A review is done on numerical analysis of furnace. (N C Mehta *et al*, April 2015). Thermal fatigue analysis of induction furnace wall is done for zirconia. (Nirajkumar C Mehta *et al*, April 2015). Comparison of finite difference method and finite element method is done for 2 D transient heat transfer problem. (Nirajkumar Mehta *et al*, April 2015). Thermal fatigue analysis of induction furnace wall is done for magnesia ramming mass. (Nirajkumar C Mehta *et al*, June 2015). Advanced mathematical modelling of heat transfer is done for induction furnace wall of zirconia. (Nirajkumar C Mehta *et al*, December 2016). Advanced heat transfer analysis is done for alumina based refractory wall of induction furnace. (Nirajkumar Mehta *et al*, December 2016).

Here, Explicit Finite Difference Method is used to find out temperature and thermal stress variation with respect to time.

2. Development of Advanced Heat Transfer Model

We have divided Induction Furnace Wall into a Nodal Network as shown in Fig. 1. It is divided into 24 nodes. We have derived Explicit Finite Difference Equations for all nodes as per the boundary conditions applied to it. The furnace wall is having thermal conduction heat transfer between different nodes. It is having atmospheric heat convection ha applied from top side of the furnace wall which is open to atmosphere. It is having heat convection from molten metal from inside which is hi. It is having heat convection ho from cooling water which is circulating outside the furnace wall. (Yunus A Cengel, Heat and Mass Transfer).

To solve this advanced heat transfer problem of induction melting furnace wall which is made from Silica Ramming Mass, the following initial and boundary conditions, material properties and basic assumptions are made:

- Refractory Materials for induction furnace wall meets the basic assumptions in the science of mechanics.
- Environmental Temperature is homogeneous at 27° C.
- Ignore the influence of heat radiation.
- Ignore the effect of gravity field.
- The surface of induction melting furnace wall is clean.
- The initial temperature of the induction melting furnace is set 27° C and it is agreement with the ambient temperature during solving the problem.
- Heat convections are considered constant for this analysis.
- Scarp material input inside furnace is considered uniform for our analysis.

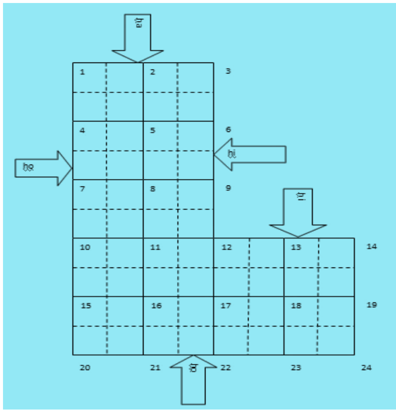


Fig.1 Nodal network for finite difference method

Node 1:

$$ha\frac{\Delta x}{2}(T_\infty - T_1^i) + ho\frac{\Delta y}{2}(T_\infty - T_1^i) + k\frac{\Delta y}{2}\frac{T_2^i - T_1^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_4^i - T_1^i}{\Delta y} = \rho\frac{\Delta x}{2}\frac{\Delta y}{2}C\frac{T_1^{i+1} - T_1^i}{\Delta t}$$

$$T_1^{i+1} = ((ha\frac{\Delta x}{2}(T_\infty - T_1^i) + ho\frac{\Delta y}{2}(T_\infty - T_1^i) + k\frac{\Delta y}{2}\frac{T_2^i - T_1^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_4^i - T_1^i}{\Delta y})\frac{4\Delta t}{\rho C\Delta x\Delta y}) + T_1^i$$

Node 2:

$$ha\Delta x(T_\infty - T_2^i) + k\frac{\Delta y}{2}\frac{T_1^i - T_2^i}{\Delta x} + k\frac{\Delta y}{2}\frac{T_3^i - T_2^i}{\Delta x} + k\Delta x\frac{T_5^i - T_2^i}{\Delta y} = \rho\Delta x\frac{\Delta y}{2}C\frac{T_2^{i+1} - T_2^i}{\Delta t}$$

$$T_2^{i+1} = ((ha\Delta x(T_\infty - T_2^i) + k\frac{\Delta y}{2}\frac{T_1^i - T_2^i}{\Delta x} + k\frac{\Delta y}{2}\frac{T_3^i - T_2^i}{\Delta x} + k\Delta x\frac{T_5^i - T_2^i}{\Delta y})\frac{2\Delta t}{\rho C\Delta x\Delta y}) + T_2^i$$

Node 3:

$$ha\frac{\Delta x}{2}(T_\infty - T_3^i) + hi\frac{\Delta y}{2}(T_h - T_3^i) + k\frac{\Delta y}{2}\frac{T_2^i - T_3^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_6^i - T_3^i}{\Delta y} = \rho\frac{\Delta x}{2}\frac{\Delta y}{2}C\frac{T_3^{i+1} - T_3^i}{\Delta t}$$

$$T_3^{i+1} = ((ha\frac{\Delta x}{2}(T_\infty - T_3^i) + hi\frac{\Delta y}{2}(T_h - T_3^i) + k\frac{\Delta y}{2}\frac{T_2^i - T_3^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_6^i - T_3^i}{\Delta y})\frac{4\Delta t}{\rho C\Delta x\Delta y}) + T_3^i$$

Node 4:

$$ho\Delta y(T_\infty - T_4^i) + k\Delta y\frac{T_5^i - T_4^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_1^i - T_4^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_7^i - T_4^i}{\Delta y} = \rho\frac{\Delta x}{2}\Delta y C\frac{T_4^{i+4} - T_4^i}{\Delta t}$$

$$T_4^{i+4} = ((ho\Delta y(T_\infty - T_4^i) + k\Delta y\frac{T_5^i - T_4^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_1^i - T_4^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_7^i - T_4^i}{\Delta y})\ \frac{2\Delta t}{\rho C\Delta x\Delta y}) + T_4^i$$

Node 5:

$$k\Delta y\frac{T_4^i - T_5^i}{\Delta x} + k\Delta y\frac{T_6^i - T_5^i}{\Delta x} + k\Delta x\frac{T_2^i - T_5^i}{\Delta y} + k\Delta x\frac{T_8^i - T_5^i}{\Delta y} = \rho\Delta x\Delta y C\frac{T_5^{i+1} - T_5^i}{\Delta t}$$

$$T_5^{i+1} = ((k\Delta y\frac{T_4^i - T_5^i}{\Delta x} + k\Delta y\frac{T_6^i - T_5^i}{\Delta x} + k\Delta x\frac{T_2^i - T_5^i}{\Delta y} + k\Delta x\frac{T_8^i - T_5^i}{\Delta y})\ \frac{\Delta t}{\rho C\Delta x\Delta y}) + T_5^i$$

Node 6:

$$hi\Delta y(T_h - T_6^i) + k\Delta y\frac{T_2^i - T_6^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_3^i - T_6^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_9^i - T_6^i}{\Delta y} = \rho\frac{\Delta x}{2}\Delta y C\frac{T_6^{i+1} - T_6^i}{\Delta t}$$

$$T_6^{i+1} = ((\ hi\Delta y(T_h - T_6^i) + k\Delta y\frac{T_2^i - T_6^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_3^i - T_6^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_9^i - T_6^i}{\Delta y})\ \frac{2\Delta t}{\rho C\Delta x\Delta y}) + T_6^i$$

Node 7:

$$ho\Delta y(T_\infty - T_7^i) + k\Delta y\frac{T_8^i - T_7^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_4^i - T_7^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{10}^i - T_7^i}{\Delta y} = \rho\frac{\Delta x}{2}\Delta y C\frac{T_7^{i+7} - T_7^i}{\Delta t}$$

$$T_7^{i+7} = ((\ ho\Delta y(T_\infty - T_7^i) + k\Delta y\frac{T_8^i - T_7^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_4^i - T_7^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{10}^i - T_7^i}{\Delta y})\ \frac{2\Delta t}{\rho C\Delta x\Delta y}) + T_7^i$$

Node 8:

$$k\Delta y\frac{T_7^i - T_8^i}{\Delta x} + k\Delta y\frac{T_9^i - T_8^i}{\Delta x} + k\Delta x\frac{T_5^i - T_8^i}{\Delta y} + k\Delta x\frac{T_{11}^i - T_8^i}{\Delta y} = \rho\Delta x\Delta y C\frac{T_8^{i+1} - T_8^i}{\Delta t}$$

$$T_8^{i+1} = ((\ k\Delta y\frac{T_7^i - T_8^i}{\Delta x} + k\Delta y\frac{T_9^i - T_8^i}{\Delta x} + k\Delta x\frac{T_5^i - T_8^i}{\Delta y} + k\Delta x\frac{T_{11}^i - T_8^i}{\Delta y})\ \frac{\Delta t}{\rho C\Delta x\Delta y}) + T_8^i$$

Node 9:

$$hi\Delta y(T_h - T_9^i) + k\Delta y\frac{T_8^i - T_9^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_6^i - T_9^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{12}^i - T_9^i}{\Delta y} = \rho\frac{\Delta x}{2}\Delta y C\frac{T_9^{i+1} - T_9^i}{\Delta t}$$

$$T_9^{i+1} = ((hi\Delta y(T_h - T_9^i) + k\Delta y\frac{T_8^i - T_9^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_6^i - T_9^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{12}^i - T_9^i}{\Delta y})\frac{2\Delta t}{\rho C\Delta x\Delta y}) + T_9^i$$

Node 10:

$$ho\Delta y(T_\infty - T_{10}^i) + k\Delta y\frac{T_{11}^i - T_{10}^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_7^i - T_{10}^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{15}^i - T_{10}^i}{\Delta y} = \rho\frac{\Delta x}{2}\Delta y C\frac{T_{10}^{i+10} - T_{10}^i}{\Delta t}$$

$$T_{10}^{i+10} = ((ho\Delta y(T_\infty - T_{10}^i) + k\Delta y\frac{T_{11}^i - T_{10}^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_7^i - T_{10}^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{15}^i - T_{10}^i}{\Delta y})\ \frac{2\Delta t}{\rho C\Delta x\Delta y}) + T_{10}^i$$

Node 11:

$$k\Delta y\frac{T_{10}^i - T_{11}^i}{\Delta x} + k\Delta y\frac{T_{12}^i - T_{11}^i}{\Delta x} + k\Delta x\frac{T_8^i - T_{11}^i}{\Delta y} + k\Delta x\frac{T_{16}^i - T_{11}^i}{\Delta y} = \rho\Delta x\Delta y C\frac{T_{11}^{i+1} - T_{11}^i}{\Delta t}$$

$$T_{11}^{i+1} = ((\ k\Delta y\frac{T_{10}^i - T_{11}^i}{\Delta x} + k\Delta y\frac{T_{12}^i - T_{11}^i}{\Delta x} + k\Delta x\frac{T_8^i - T_{11}^i}{\Delta y} + k\Delta x\frac{T_{16}^i - T_{11}^i}{\Delta y})\ \frac{\Delta t}{\rho C\Delta x\Delta y}) + T_{11}^i$$

Node 12:

$$hi\frac{\Delta x}{2}(T_h - T_{12}^i) + hi\frac{\Delta y}{2}(T_h - T_{12}^i) + k\Delta y\frac{T_{11}^i - T_{12}^i}{\Delta x} + k\frac{\Delta y}{2}\frac{T_{13}^i - T_{12}^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_9^i - T_{12}^i}{\Delta y} + k\Delta x\frac{T_{17}^i - T_{12}^i}{\Delta y} = \rho\frac{3\Delta x\Delta y}{4}C\frac{T_{12}^{i+1} - T_{12}^i}{\Delta t}$$

$$T_{12}^{i+1} = ((\ hi\frac{\Delta x}{2}(T_h - T_{12}^i) + hi\frac{\Delta y}{2}(T_h - T_{12}^i) + k\Delta y\frac{T_{11}^i - T_{12}^i}{\Delta x} + k\frac{\Delta y}{2}\frac{T_{13}^i - T_{12}^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_9^i - T_{12}^i}{\Delta y} + k\Delta x\frac{T_{17}^i - T_{12}^i}{\Delta y})\ \frac{4\Delta t}{3\rho C\Delta x\Delta y}) + T_{12}^i$$

Node 13:

$$hi\Delta x(T_h - T_{13}^i) + k\frac{\Delta y}{2}\frac{T_{12}^i - T_{13}^i}{\Delta x} + k\frac{\Delta y}{2}\frac{T_{14}^i - T_{13}^i}{\Delta x} + k\Delta x\frac{T_{18}^i - T_{13}^i}{\Delta y} = \rho\Delta x\frac{\Delta y}{2}C\frac{T_{13}^{i+1} - T_{13}^i}{\Delta t}$$

$$T_{13}^{i+1} = ((\ hi\Delta x(T_h - T_{13}^i) + k\frac{\Delta y}{2}\frac{T_{12}^i - T_{13}^i}{\Delta x} + k\frac{\Delta y}{2}\frac{T_{14}^i - T_{13}^i}{\Delta x} + k\Delta x\frac{T_{18}^i - T_{13}^i}{\Delta y})\ \frac{2\Delta t}{\rho C\Delta x\Delta y}) + T_{13}^i$$

Node 14:

$$hi\frac{\Delta x}{2}(T_h - T_{14}^i) + hi\frac{\Delta y}{2}(T_h - T_{14}^i) + k\frac{\Delta y}{2}\frac{T_{13}^i - T_{14}^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_{19}^i - T_{14}^i}{\Delta y} = \rho\frac{\Delta x}{2}\frac{\Delta y}{2}C\frac{T_{14}^{i+1} - T_{14}^i}{\Delta t}$$

$$T_{14}^{i+1} = ((\ hi\frac{\Delta x}{2}(T_h - T_{14}^i) + hi\frac{\Delta y}{2}(T_h - T_{14}^i) + k\frac{\Delta y}{2}\frac{T_{13}^i - T_{14}^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_{19}^i - T_{14}^i}{\Delta y})\ \frac{4\Delta t}{\rho C\Delta x\Delta y}) + T_{14}^i$$

Node 15:

$$ho\Delta y(T_\infty - T_{15}^i) + k\Delta y\frac{T_{16}^i - T_{15}^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_{10}^i - T_{15}^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{20}^i - T_{15}^i}{\Delta y} = \rho\frac{\Delta x}{2}\Delta y C\frac{T_{15}^{i+15} - T_{15}^i}{\Delta t}$$

$$T_{15}^{i+15} = ((\ ho\Delta y(T_\infty - T_{15}^i) + k\Delta y\frac{T_{16}^i - T_{15}^i}{\Delta x} + k\frac{\Delta x}{2}\frac{T_{10}^i - T_{15}^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_{20}^i - T_{15}^i}{\Delta y})\ \frac{2\Delta t}{\rho C\Delta x\Delta y}) + T_{15}^i$$

Node 16:

$$k\Delta y\frac{T_{15}^i - T_{16}^i}{\Delta x} + k\Delta y\frac{T_{17}^i - T_{16}^i}{\Delta x} + k\Delta x\frac{T_{11}^i - T_{16}^i}{\Delta y} + k\Delta x\frac{T_{21}^i - T_{16}^i}{\Delta y} = \rho\Delta x\Delta y C\frac{T_{16}^{i+1} - T_{16}^i}{\Delta t}$$

$$T_{16}^{i+1} = ((\ k\Delta y\frac{T_{15}^i - T_{16}^i}{\Delta x} + k\Delta y\frac{T_{17}^i - T_{16}^i}{\Delta x} + k\Delta x\frac{T_{11}^i - T_{16}^i}{\Delta y} + k\Delta x\frac{T_{21}^i - T_{16}^i}{\Delta y})\ \frac{\Delta t}{\rho C\Delta x\Delta y}) + T_{16}^i$$

Node 17:

$$k\Delta y\frac{T_{16}^{i}-T_{17}^{i}}{\Delta x}+k\Delta y\frac{T_{18}^{i}-T_{17}^{i}}{\Delta x}+k\Delta x\frac{T_{12}^{i}-T_{17}^{i}}{\Delta y}+k\Delta x\frac{T_{22}^{i}-T_{17}^{i}}{\Delta y}=\rho\Delta x\Delta yC\frac{T_{17}^{i+1}-T_{17}^{i}}{\Delta t}$$

$$T_{17}^{i+1}=((\;k\Delta y\frac{T_{16}^{i}-T_{17}^{i}}{\Delta x}+k\Delta y\frac{T_{18}^{i}-T_{17}^{i}}{\Delta x}+k\Delta x\frac{T_{12}^{i}-T_{17}^{i}}{\Delta y}+k\Delta x\frac{T_{22}^{i}-T_{17}^{i}}{\Delta y})\;\frac{\Delta t}{\rho C\Delta x\Delta y})+T_{17}^{i}$$

Node 18:

$$k\Delta y\frac{T_{17}^{i}-T_{18}^{i}}{\Delta x}+k\Delta y\frac{T_{19}^{i}-T_{18}^{i}}{\Delta x}+k\Delta x\frac{T_{13}^{i}-T_{18}^{i}}{\Delta y}+k\Delta x\frac{T_{23}^{i}-T_{18}^{i}}{\Delta y}=\rho\Delta x\Delta yC\frac{T_{18}^{i+1}-T_{18}^{i}}{\Delta t}$$

$$T_{18}^{i+1}=((k\Delta y\frac{T_{17}^{i}-T_{18}^{i}}{\Delta x}+k\Delta y\frac{T_{19}^{i}-T_{18}^{i}}{\Delta x}+k\Delta x\frac{T_{13}^{i}-T_{18}^{i}}{\Delta y}+k\Delta x\frac{T_{23}^{i}-T_{18}^{i}}{\Delta y})\;\frac{\Delta t}{\rho C\Delta x\Delta y})+T_{18}^{i}$$

Node 19:

$$hi\Delta y(T_h-T_{19}^{i})+k\Delta y\frac{T_{18}^{i}-T_{19}^{i}}{\Delta x}+k\frac{\Delta x}{2}\frac{T_{14}^{i}-T_{19}^{i}}{\Delta y}+k\frac{\Delta x}{2}\frac{T_{24}^{i}-T_{19}^{i}}{\Delta y}=\rho\frac{\Delta x}{2}\Delta yC\frac{T_{19}^{i+1}-T_{19}^{i}}{\Delta t}$$

$$T_{19}^{i+1}=((hi\Delta y(T_h-T_{19}^{i})+k\Delta y\frac{T_{18}^{i}-T_{19}^{i}}{\Delta x}+k\frac{\Delta x}{2}\frac{T_{14}^{i}-T_{19}^{i}}{\Delta y}+k\frac{\Delta x}{2}\frac{T_{24}^{i}-T_{19}^{i}}{\Delta y})\;\frac{2\Delta t}{\rho C\Delta x\Delta y})+T_{19}^{i}$$

Node 20:

$$ho\frac{\Delta x}{2}(T_\infty-T_{20}^{i})+ho\frac{\Delta y}{2}(T_\infty-T_{20}^{i})+k\frac{\Delta y}{2}\frac{T_{21}^{i}-T_{20}^{i}}{\Delta x}+k\frac{\Delta x}{2}\frac{T_{15}^{i}-T_{20}^{i}}{\Delta y}=\rho\frac{\Delta x}{2}\frac{\Delta y}{2}C\frac{T_{20}^{i+20}-T_{20}^{i}}{\Delta t}$$

$$T_{20}^{i+20}=((ho\frac{\Delta x}{2}(T_\infty-T_{20}^{i})+ho\frac{\Delta y}{2}(T_\infty-T_{20}^{i})+k\frac{\Delta y}{2}\frac{T_{21}^{i}-T_{20}^{i}}{\Delta x}+k\frac{\Delta x}{2}\frac{T_{15}^{i}-T_{20}^{i}}{\Delta y})\;\frac{4\Delta t}{\rho C\Delta x\Delta y})+T_{20}^{i}$$

Node 21:

$$ho\Delta x(T_\infty-T_{21}^{i})+k\frac{\Delta y}{2}\frac{T_{20}^{i}-T_{21}^{i}}{\Delta x}+k\frac{\Delta y}{2}\frac{T_{22}^{i}-T_{21}^{i}}{\Delta x}+k\Delta x\frac{T_{16}^{i}-T_{21}^{i}}{\Delta y}=\rho\Delta x\frac{\Delta y}{2}C\frac{T_{21}^{i+1}-T_{21}^{i}}{\Delta t}$$

$$T_{21}^{i+1}=((ho\Delta x(T_\infty-T_{21}^{i})+k\frac{\Delta y}{2}\frac{T_{20}^{i}-T_{21}^{i}}{\Delta x}+k\frac{\Delta y}{2}\frac{T_{22}^{i}-T_{21}^{i}}{\Delta x}+k\Delta x\frac{T_{16}^{i}-T_{21}^{i}}{\Delta y})\;\frac{2\Delta t}{\rho C\Delta x\Delta y})+T_{21}^{i}$$

Node 22:

$$ho\Delta x(T_\infty-T_{22}^{i})+k\frac{\Delta y}{2}\frac{T_{21}^{i}-T_{22}^{i}}{\Delta x}+k\frac{\Delta y}{2}\frac{T_{23}^{i}-T_{22}^{i}}{\Delta x}+k\Delta x\frac{T_{17}^{i}-T_{22}^{i}}{\Delta y}=\rho\Delta x\frac{\Delta y}{2}C\frac{T_{22}^{i+1}-T_{22}^{i}}{\Delta t}$$

$$T_{22}^{i+1}=((ho\Delta x(T_\infty-T_{22}^{i})+k\frac{\Delta y}{2}\frac{T_{21}^{i}-T_{22}^{i}}{\Delta x}+k\frac{\Delta y}{2}\frac{T_{23}^{i}-T_{22}^{i}}{\Delta x}+k\Delta x\frac{T_{17}^{i}-T_{22}^{i}}{\Delta y})\;\frac{2\Delta t}{\rho C\Delta x\Delta y})+T_{22}^{i}$$

Node 23:

$$ho\Delta x(T_\infty-T_{23}^{i})+k\frac{\Delta y}{2}\frac{T_{22}^{i}-T_{23}^{i}}{\Delta x}+k\frac{\Delta y}{2}\frac{T_{24}^{i}-T_{23}^{i}}{\Delta x}+k\Delta x\frac{T_{18}^{i}-T_{23}^{i}}{\Delta y}=\rho\Delta x\frac{\Delta y}{2}C\frac{T_{23}^{i+1}-T_{23}^{i}}{\Delta t}$$

$$T_{23}^{i+1}=((ho\Delta x(T_\infty-T_{23}^{i})+k\frac{\Delta y}{2}\frac{T_{22}^{i}-T_{23}^{i}}{\Delta x}+k\frac{\Delta y}{2}\frac{T_{24}^{i}-T_{23}^{i}}{\Delta x}+k\Delta x\frac{T_{18}^{i}-T_{23}^{i}}{\Delta y})\;\frac{2\Delta t}{\rho C\Delta x\Delta y})+T_{23}^{i}$$

Node 24:

$$ho\frac{\Delta x}{2}(T_\infty-T_{24}^{i})+hi\frac{\Delta y}{2}(T_h-T_{24}^{i})+k\frac{\Delta y}{2}\frac{T_{23}^{i}-T_{24}^{i}}{\Delta x}+k\frac{\Delta x}{2}\frac{T_{19}^{i}-T_{24}^{i}}{\Delta y}=\rho\frac{\Delta x}{2}\frac{\Delta y}{2}C\frac{T_{24}^{i+1}-T_{24}^{i}}{\Delta t}$$

$$T_{24}^{i+1}=((ho\frac{\Delta x}{2}(T_\infty-T_{24}^{i})+hi\frac{\Delta y}{2}(T_h-T_{24}^{i})+k\frac{\Delta y}{2}\frac{T_{23}^{i}-T_{24}^{i}}{\Delta x}+k\frac{\Delta x}{2}\frac{T_{19}^{i}-T_{24}^{i}}{\Delta y})\;\frac{4\Delta t}{\rho C\Delta x\Delta y})+T_{24}^{i}$$

3. Programming & Solution

With the help of a computer program we can solve the matrix created by finite difference equations for 24 nodes. We can calculate temperature distribution and stress distribution with respect to time.

Table 1 Material Property and Boundary Conditions

Material Properties and Boundary Conditions for Silica Ramming Mass			Unit
1	Internal Film Co-efficient hi	200	W/m² K
2	External Film Co-efficient ho	40	W/m² K
3	Atmosphere Film Co-efficient ha	10	W/m² K
4	Density	2800	Kg/m³
5	Time Interval Δt	10	Seconds
6	Thermal Conductivity k	1.7	W/m K
7	Temperature outside Furnace Wall	303	Kelvin
8	Temperature inside Furnace Wall	1673	Kelvin
9	Temperature of Air	303	Kelvin
10	Specific Heat	950	J/kg K
11	Elasticity Constant	200000	N/ m²
12	Thermal Expansion Co-efficient	0.00000122	m/ K
13	Ultimate Stress	500	MPa

Computer Program

```
#include<iostream.h>
#include<conio.h>
#include<math.h>
#include<complex.h>
#include<fstream.h>
void main()
{     clrscr;
ofstreammyfile;
myfile.open ("outputfdm.txt");
myfile<<"Writing this to a file.\n";
float T[25][500];
float S[25][500];
float hi,ho,ha,To,r,t,k,Ta,c,Th,E,Af,Sut,Se,m,N,L;
cout<<"Enter in W/m2 K value of hi";
cin>>hi;
cout<<"Enter in W/m2 K value of ho";
cin>>ho;
cout<<"Enter in W/m2 K value of ha";
cin>>ha;
cout<<"Enter in Kg/m3 value of density";
cin>>r;
cout<<"Enter in seconds time interval delta t";
cin>>t;
cout<<"Enter in W/m K value of thermal conductivity";
```

```
cin>>k;
cout<<"Enter in Kelvin Temperatute outside Furnace Wall";
cin>>Ta;
cout<<"Enter in Kelvin Temperature inside Furnace Wall";
cin>>Th;
cout<<"Enter in Kelvin Temperature of Air";
cin>>To;
cout<<"Enter in J/Kg K value of Specific Heat";
cin>>c;
cout<<"Enter in N/m2 value of Elasticity Constant";
cin>>E;
cout<<"Enter in m/K Thermal Expansion Co-efficient";
cin>>Af;
cout<<"Enter in MPA value of Ultimate Stress";
cin>>Sut;
T[1][0]=300;
T[2][0]=300;
T[3][0]=300;
T[4][0]=300;
T[5][0]=300;
T[6][0]=300;
T[7][0]=300;
T[8][0]=300;
T[9][0]=300;
T[10][0]=300;
T[11][0]=300;
T[12][0]=300;
T[13][0]=300;
T[14][0]=300;
T[15][0]=300;
T[16][0]=300;
T[17][0]=300;
T[18][0]=300;
T[19][0]=300;
T[20][0]=300;
T[21][0]=300;
T[22][0]=300;
T[23][0]=300;
T[24][0]=300;
for (inti=0; i<=270;i++)
{
T[1][i+1] = (((0.5*ha*0.06*(To-T[1][i])) +(ho*0.3*(Ta-T[1][i])/2)+(0.5*k*0.3*(T[2][i]-T[1][i])/0.06)+(0.5*k*0.06*(T[4][i]-T[1][i])/0.3) )*((4*t)/(r*c*0.06*0.3)))+T[1][i];
T[2][i+1] = (((ha*0.06*(To-T[2][i]))+(0.5*k*0.3*(T[1][i]-T[2][i])/0.06)+(0.5*k*0.3*(T[3][i]-T[2][i])/0.06) +(0.5*k*0.06*(T[5][i]-T[2][i])/0.3))*((2*t)/(r*c*0.06*0.3)))+T[2][i];
T[3][i+1] = (((ha*0.06*(To-T[3][i])*0.5) +( hi*0.3*(Th-T[3][i])*0.5 ) +(0.5*k*0.3*(T[2][i]-T[3][i])/0.06)+(0.5*k*0.06*(T[6][i]-T[3][i])/0.3)) *((4*t)/(r*c*0.06*0.3)))+T[3][i];
T[4][i+1] = ((( ho*0.3*(Ta-T[4][i])) +(k*0.3*(T[5][i]-T[4][i])/0.06)+(0.5*k*0.06*(T[1][i]-T[4][i])/0.3) + (0.5*k*0.06*(T[7][i]-T[4][i])/0.3)) *((2*t)/(r*c*0.06*0.3)))+T[4][i];
T[5][i+1] = (((k*0.3*(T[4][i]-T[5][i])/0.06)+(k*0.3*(T[6][i]-T[5][i])/0.06) + (k*0.06*(T[8][i]-T[5][i])/0.3)) *((t)/(r*c*0.06*0.3)))+T[5][i];
T[6][i+1] = (((hi*0.3*(Th-T[6][i])) + (k*0.3*(T[2][i]-T[6][i])/0.06)+(0.5*k*0.06*(T[3][i]-T[6][i])/0.3) + (0.5*k*0.06*(T[9][i]-T[6][i])/0.3)) *((2*t)/(r*c*0.06*0.3)))+T[6][i];
T[7][i+1] = (((ho*0.3*(Ta-T[7][i])) + (k*0.3*(T[8][i]-T[7][i])/0.06)+(0.5*k*0.06*(T[4][i]-T[7][i])/0.3) + (0.5*k*0.06*(T[10][i]-T[7][i])/0.3)) *((2*t)/(r*c*0.06*0.3)))+T[7][i];
T[8][i+1] = (((k*0.3*(T[7][i]-T[8][i])/0.06)+( k*0.3*(T[9][i]-T[8][i])/0.06 ) +(k*0.06*(T[5][i]-T[8][i])/0.3) + (k*0.06*(T[11][i]-T[8][i])/0.3)) *((t)/(r*c*0.06*0.3)))+T[8][i];
T[9][i+1] = (( (hi*0.3*(Th-T[9][i]))+(k*0.3*(T[8][i]-T[9][i])/0.06)+ (0.5*k*0.06*(T[6][i]-T[9][i])/0.3) + (0.5*k*0.06*(T[12][i]-T[9][i])/0.3)) *((2*t)/(r*c*0.06*0.3)))+T[9][i];
T[10][i+1] = (( (ho*0.18*(Ta-T[10][i]))+(k*0.18*(T[11][i]-T[10][i])/0.06)+ (0.5*k*0.06*(T[7][i]-T[10][i])/0.18) + (0.5*k*0.06*(T[15][i]-T[10][i])/0.18)) *((2*t)/(r*c*0.06*0.18)))+T[10][i];

T[11][i+1] = (( (k*0.18*(T[10][i]-T[11][i])/0.06)+ (k*0.18*(T[12][i]-T[11][i])/0.06)+ (k*0.06*(T[8][i]-T[11][i])/0.18) + (k*0.06*(T[16][i]-T[11][i])/0.18)) *((t)/(r*c*0.06*0.18)))+T[11][i];
T[12][i+1] = (( (0.5*hi*0.12*(Th-T[12][i]))+( 0.5*hi*0.18*(Th-T[12][i])) +(k*0.18*(T[11][i]-T[12][i])/0.12)+ (0.5*k*0.18*(T[13][i]-T[12][i])/0.12)+(0.5*k*0.12*(T[9][i]-T[12][i])/0.18) + (k*0.12*(T[17][i]-T[12][i])/0.18)) *((4*t)/(3*r*c*0.12*0.18))) +T[12][i];
T[13][i+1] = (( (hi*0.18*(Th-T[13][i]))+(0.5*k*0.18*(T[12][i]-T[13][i])/0.18)+ (0.5*k*0.18*(T[14][i]-T[13][i])/0.18)+ (k*0.18*(T[18][i]-T[13][i])/0.18)) *((2*t)/(r*c*0.18*0.18)))+T[13][i];
T[14][i+1] = (( (0.5*hi*0.18*(Th-T[14][i]))+(0.5*hi*0.18*(Th-T[14][i])) +(0.5*k*0.18*(T[13][i]-T[14][i])/0.18)+ (0.5*k*0.18*(T[19][i]-T[14][i])/0.18)) *((4*t)/(r*c*0.18*0.18)))+T[14][i];
T[15][i+1] = (( (ho*0.06*(Ta-T[15][i]))+(k*0.06*(T[16][i]-T[15][i])/0.06)+ (0.5*k*0.06*(T[10][i]-T[15][i])/0.06)+( 0.5*k*0.06*(T[20][i]-T[15][i])/0.06)) *((2*t)/(r*c*0.06*0.06)))+T[15][i];
T[16][i+1] = (( (k*0.06*(T[15][i]-T[16][i])/0.06)+ (k*0.06*(T[17][i]-T[16][i])/0.06 )+(k*0.06*(T[11][i]-T[16][i])/0.06)+( k*0.06*(T[21][i]-T[16][i])/0.06)) *((t)/(r*c*0.06*0.06)))+T[16][i];
T[17][i+1] = (( (k*0.06*(T[16][i]-T[17][i])/0.12)+ (k*0.06*(T[18][i]-T[17][i])/0.12 )+(k*0.12*(T[12][i]-T[17][i])/0.06)+( k*0.12*(T[22][i]-T[17][i])/0.06)) *((t)/(r*c*0.12*0.06)))+T[17][i];
T[18][i+1] = (( (k*0.06*(T[17][i]-T[18][i])/0.18)+ (k*0.06*(T[19][i]-T[18][i])/0.18 )+(k*0.18*(T[13][i]-
```

```
T[18][i])/0.06)+( k*0.18*(T[23][i]-T[18][i])/0.06))
*((t)/(r*c*0.18*0.06)))+T[18][i];
T[19][i+1] = (( (hi*0.06*(Th-
T[19][i]))+(k*0.06*(T[18][i]-T[19][i])/0.18)+
(0.5*k*0.18*(T[14][i]-T[19][i])/0.06)+(
0.5*k*0.18*(T[24][i]-T[19][i])/0.06))
*((2*t)/(r*c*0.18*0.06)))+T[19][i];
T[20][i+1] = (( (0.5*ho*0.06*(Ta-T[20][i]))+(
0.5*ho*0.06*(Ta-T[20][i]))+(0.5*k*0.06*(T[21][i]-
T[20][i])/0.06)+ (0.5*k*0.06*(T[15][i]-
T[20][i])/0.06))*((4*t)/(r*c*0.06*0.06)))+T[20][i];

T[21][i+1] = (( (0.5*ho*0.06*(Ta-
T[21][i]))+(0.5*k*0.06*(T[20][i]-T[21][i])/0.06)+(
0.5*k*0.06*(T[22][i]-T[21][i])/0.06)+
(k*0.06*(T[16][i]-
T[21][i])/0.06))*((2*t)/(r*c*0.06*0.06)))+T[21][i];
T[22][i+1] = (( (ho*0.12*(Ta-
T[22][i]))+(0.5*k*0.06*(T[21][i]-T[22][i])/0.12)+(
0.5*k*0.06*(T[23][i]-T[22][i])/0.12)+
(k*0.12*(T[17][i]-
T[22][i])/0.06))*((2*t)/(r*c*0.12*0.06)))+T[22][i];
T[23][i+1] = (( (ho*0.18*(Ta-
T[23][i]))+(0.5*k*0.06*(T[22][i]-T[23][i])/0.18)+(
0.5*k*0.06*(T[24][i]-T[23][i])/0.18)+
(k*0.18*(T[18][i]-
T[23][i])/0.06))*((2*t)/(r*c*0.18*0.06)))+T[23][i];
T[24][i+1] = (( (0.5*ho*0.18*(Ta-T[24][i]))+(
0.5*hi*0.06*(Th-T[24][i])) +(0.5*k*0.06*(T[23][i]-
T[24][i])/0.18)+ (0.5*k*0.18*(T[19][i]-
T[24][i])/0.06))*((4*t)/(r*c*0.18*0.06)))+T[24][i];
S[14][i] = E*Af*(T[14][i]-Ta);
}
for (i=271; i<=360;i++)
{
hi=ha;
Th=Ta;
T[1][i+1] = (((0.5*ha*0.06*(To-T[1][i])) +(ho*0.3*(Ta-
T[1][i])/2)+(0.5*k*0.3*(T[2][i]-
T[1][i])/0.06)+(0.5*k*0.06*(T[4][i]-T[1][i])/0.3)
)*((4*t)/(r*c*0.06*0.3)))+T[1][i];
T[2][i+1] = (((ha*0.06*(To-
T[2][i]))+(0.5*k*0.3*(T[1][i]-
T[2][i])/0.06)+(0.5*k*0.3*(T[3][i]-T[2][i])/0.06)
+(0.5*k*0.06*(T[5][i]-
T[2][i])/0.3))*((2*t)/(r*c*0.06*0.3)))+T[2][i];
T[3][i+1] = (((ha*0.06*(To-T[3][i])*0.5) +( hi*0.3*(Th-
T[3][i])*0.5 ) +(0.5*k*0.3*(T[2][i]-
T[3][i])/0.06)+(0.5*k*0.06*(T[6][i]-T[3][i])/0.3))
*((4*t)/(r*c*0.06*0.3)))+T[3][i];
T[4][i+1] = ((( ho*0.3*(Ta-T[4][i])) +(k*0.3*(T[5][i]-
T[4][i])/0.06)+(0.5*k*0.06*(T[1][i]-T[4][i])/0.3) +
(0.5*k*0.06*(T[7][i]-T[4][i])/0.3))
*((2*t)/(r*c*0.06*0.3)))+T[4][i];
T[5][i+1] = (((k*0.3*(T[4][i]-
T[5][i])/0.06)+(k*0.3*(T[6][i]-T[5][i])/0.06) +
(k*0.06*(T[8][i]-T[5][i])/0.3))
*((t)/(r*c*0.06*0.3)))+T[5][i];
T[6][i+1] = (((hi*0.3*(Th-T[6][i])) + (k*0.3*(T[2][i]-
T[6][i])/0.06)+(0.5*k*0.06*(T[3][i]-T[6][i])/0.3) +
(0.5*k*0.06*(T[9][i]-T[6][i])/0.3))
*((2*t)/(r*c*0.06*0.3)))+T[6][i];
T[7][i+1] = (((ho*0.3*(Ta-T[7][i])) + (k*0.3*(T[8][i]-
T[7][i])/0.06)+(0.5*k*0.06*(T[4][i]-T[7][i])/0.3) +
(0.5*k*0.06*(T[10][i]-T[7][i])/0.3))
*((2*t)/(r*c*0.06*0.3)))+T[7][i];
T[8][i+1] = (((k*0.3*(T[7][i]-T[8][i])/0.06)+(
k*0.3*(T[9][i]-T[8][i])/0.06 ) +(k*0.06*(T[5][i]-
T[8][i])/0.3) + (k*0.06*(T[11][i]-T[8][i])/0.3))
*((t)/(r*c*0.06*0.3)))+T[8][i];
T[9][i+1] = (( (hi*0.3*(Th-T[9][i]))+(k*0.3*(T[8][i]-
T[9][i])/0.06)+ (0.5*k*0.06*(T[6][i]-T[9][i])/0.3) +
(0.5*k*0.06*(T[12][i]-T[9][i])/0.3))
*((2*t)/(r*c*0.06*0.3)))+T[9][i];
T[10][i+1] = (( (ho*0.18*(Ta-
T[10][i]))+(k*0.18*(T[11][i]-T[10][i])/0.06)+
(0.5*k*0.06*(T[7][i]-T[10][i])/0.18) +
(0.5*k*0.06*(T[15][i]-T[10][i])/0.18))
*((2*t)/(r*c*0.06*0.18)))+T[10][i];
T[11][i+1] = (( (k*0.18*(T[10][i]-T[11][i])/0.06)+
(k*0.18*(T[12][i]-T[11][i])/0.06)+ (k*0.06*(T[8][i]-
T[11][i])/0.18) + (k*0.06*(T[16][i]-T[11][i])/0.18))
*((t)/(r*c*0.06*0.18)))+T[11][i];
T[12][i+1] = (( (0.5*hi*0.12*(Th-T[12][i]))+(
0.5*hi*0.18*(Th-T[12][i])) +(k*0.18*(T[11][i]-
T[12][i])/0.12)+ (0.5*k*0.18*(T[13][i]-
T[12][i])/0.12)+(0.5*k*0.12*(T[9][i]-T[12][i])/0.18) +
(k*0.12*(T[17][i]-T[12][i])/0.18))
*((4*t)/(3*r*c*0.12*0.18))) +T[12][i];
T[13][i+1] = (( (hi*0.18*(Th-
T[13][i]))+(0.5*k*0.18*(T[12][i]-T[13][i])/0.18)+
(0.5*k*0.18*(T[14][i]-T[13][i])/0.18)+
(k*0.18*(T[18][i]-T[13][i])/0.18))
*((2*t)/(r*c*0.18*0.18)))+T[13][i];
T[14][i+1] = (( (0.5*hi*0.18*(Th-
T[14][i]))+(0.5*hi*0.18*(Th-T[14][i]))
+(0.5*k*0.18*(T[13][i]-T[14][i])/0.18)+
(0.5*k*0.18*(T[19][i]-T[14][i])/0.18))
*((4*t)/(r*c*0.18*0.18)))+T[14][i];
T[15][i+1] = (( (ho*0.06*(Ta-
T[15][i]))+(k*0.06*(T[16][i]-T[15][i])/0.06)+
(0.5*k*0.06*(T[10][i]-T[15][i])/0.06)+(
0.5*k*0.06*(T[20][i]-T[15][i])/0.06))
*((2*t)/(r*c*0.06*0.06)))+T[15][i];
T[16][i+1] = (( (k*0.06*(T[15][i]-T[16][i])/0.06)+
(k*0.06*(T[17][i]-T[16][i])/0.06 )+(k*0.06*(T[11][i]-
T[16][i])/0.06)+( k*0.06*(T[21][i]-T[16][i])/0.06))
*((t)/(r*c*0.06*0.06)))+T[16][i];

T[17][i+1] = (( (k*0.06*(T[16][i]-T[17][i])/0.12)+
(k*0.06*(T[18][i]-T[17][i])/0.12 )+(k*0.12*(T[12][i]-
T[17][i])/0.06)+( k*0.12*(T[22][i]-T[17][i])/0.06))
*((t)/(r*c*0.12*0.06)))+T[17][i];
T[18][i+1] = (( (k*0.06*(T[17][i]-T[18][i])/0.18)+
(k*0.06*(T[19][i]-T[18][i])/0.18 )+(k*0.18*(T[13][i]-
T[18][i])/0.06)+( k*0.18*(T[23][i]-T[18][i])/0.06))
*((t)/(r*c*0.18*0.06)))+T[18][i];
T[19][i+1] = (( (hi*0.06*(Th-
T[19][i]))+(k*0.06*(T[18][i]-T[19][i])/0.18)+
(0.5*k*0.18*(T[14][i]-T[19][i])/0.06)+(
0.5*k*0.18*(T[24][i]-T[19][i])/0.06))
*((2*t)/(r*c*0.18*0.06)))+T[19][i];
```

```
T[20][i+1] = (( (0.5*ho*0.06*(Ta-T[20][i]))+(
0.5*ho*0.06*(Ta-T[20][i]))+(0.5*k*0.06*(T[21][i]-
T[20][i])/0.06)+ (0.5*k*0.06*(T[15][i]-
T[20][i])/0.06))*((4*t)/(r*c*0.06*0.06)))+T[20][i];
T[21][i+1] = (( (0.5*ho*0.06*(Ta-
T[21][i]))+(0.5*k*0.06*(T[20][i]-T[21][i])/0.06)+(
0.5*k*0.06*(T[22][i]-T[21][i])/0.06)+
(k*0.06*(T[16][i]-
T[21][i])/0.06))*((2*t)/(r*c*0.06*0.06)))+T[21][i];
T[22][i+1] = (( (ho*0.12*(Ta-
T[22][i]))+(0.5*k*0.06*(T[21][i]-T[22][i])/0.12)+(
0.5*k*0.06*(T[23][i]-T[22][i])/0.12)+
(k*0.12*(T[17][i]-
T[22][i])/0.06))*((2*t)/(r*c*0.12*0.06)))+T[22][i];
T[23][i+1] = (( (ho*0.18*(Ta-
T[23][i]))+(0.5*k*0.06*(T[22][i]-T[23][i])/0.18)+(
0.5*k*0.06*(T[24][i]-T[23][i])/0.18)+
(k*0.18*(T[18][i]-
T[23][i])/0.06))*((2*t)/(r*c*0.18*0.06)))+T[23][i];
T[24][i+1] = (( (0.5*ho*0.18*(Ta-T[24][i]))+(
0.5*hi*0.06*(Th-T[24][i])) +(0.5*k*0.06*(T[23][i]-
T[24][i])/0.18)+ (0.5*k*0.18*(T[19][i]-
T[24][i])/0.06))*((4*t)/(r*c*0.18*0.06)))+T[24][i];
S[14][i] = E*Af*(T[14][i]-Ta);
 }
for(i=0; i<=360;i=i+6)
{
myfile<<"MAXIMUM TEMPERATURE AFTER
"<<i/6<<" MINUTES"<<T[14][i]<<" KELVIN"<<endl;
myfile<<"MAXIMUM STRESS AFTER "<<i/6<<"
MINUTES "<<S[14][i]<<" MPa"<<endl; }
for(i=0; i<=300; i++)
{
if (i==270)
{
myfile<<"S"<<S[14][i];
Se=0.15*Sut;
m = (Sut-Se)/7;
N = 7 -((S[14][i]-Se)/m);
L = pow(10,N);
myfile<<"N"<<N;
myfile<<"LIFE CYCLE"<<L;
myfile.close();
}
}
getch(); }
```

4. Results and Discussion

We can see from the Fig. 2 that maximum temperature is increasing from atmospheric temperature 300 K and reaches to maximum temperature 1623 K in 45 minutes and then starts reducing and reaches to 724 K in next 15 minutes. It again starts increasing and reaches to maximum 1623 K after 105 minutes and again starts reducing. There are 10 similar temperature cycles in one day.

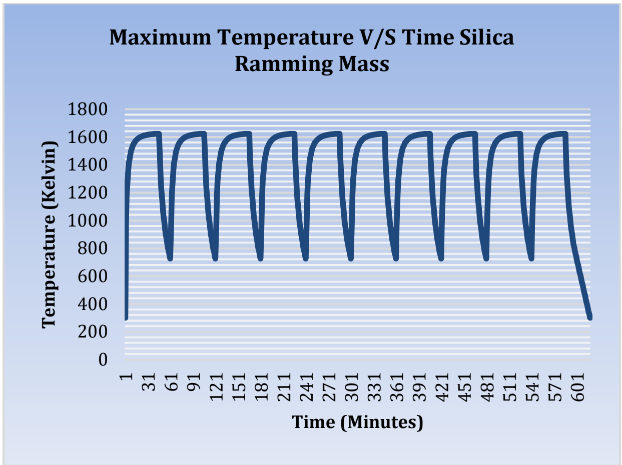


Fig.2 Maximum temperature v/s time graph for silica ramming mass

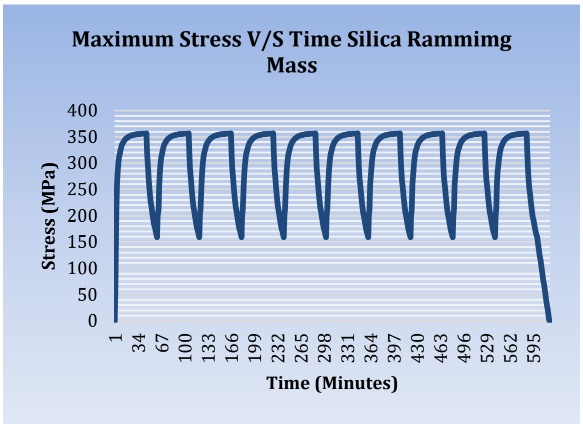


Fig.3 Maximum thermal stress v/s time graph for silica ramming mass

We can see from the Fig. 3 that maximum thermal stress is increasing from initial condition 0 MPa and reaches to maximum stress 356MPa in 45 minutes and then starts reducing and reaches to 159MPa in next 15 minutes. It again starts increasing and reaches to maximum stress 356 MPa after 105 minutes and again it starts reducing. There are 10 similar thermal stress cycles in one day.

Stress-Life Method

To determine life of any component by Stress-Life Method, we need to find out ultimate strength and endurance limit of the component for the required material.

We know the values of ultimate stress for these all materials. Silica ramming mass is having ultimate strength of 500 MPa. We can find out Se' from the equation given below or we can say dividing value of ultimate strength.

We know value of ultimate stress of silica ramming mass is 500 Mpa.

We know the relation between Sut and Se' so that we can find out Se'.

*Se' = 0.5 * Sut*= 250 MPa

Endurance Limit Modifying Factors

We have seen that the rotating-beam specimen used in the laboratory to determine endurance limits is prepared very carefully and tested under closely controlled conditions.

It is unrealistic to expect the endurance limit of a mechanical or structural member to match the values obtained in the laboratory.

Some differences include

- *Material:* composition, basis of failure, variability
- *Manufacturing:* method, heat treatment, fretting corrosion, surface condition, stress concentration
- *Environment:* corrosion, temperature, stress state, relaxation times
- *Design:* size, shape, life, stress state, stress concentration, speed, fretting, galling

Marin identified factors that quantified the effects of surface condition, size, loading, temperature, and miscellaneous items. The question of whether to adjust the endurance limit by subtractive corrections or multiplicative corrections was resolved by an extensive statistical analysis of a 4340 (electric furnace, aircraft quality) steel, in which a correlation coefficient of 0.85 was found for the multiplicative form and 0.40 for the additive form.

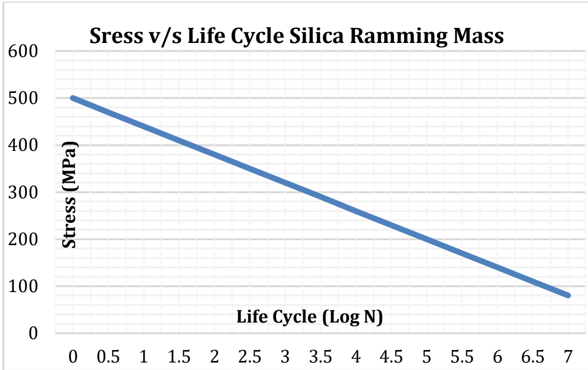
A Marin equation is therefore written as

Se = ka*kb*kc*kd*ke*kf* Se'
Where,
ka= surface condition modification factor = 0.84
kb= size modification factor = 0.98
kc= load modification factor = 0.96
kd= temperature modification factor = 0.45
ke= reliability factor = 0.9
kf= miscellaneous-effects modification factor = 0.95
Se'= specimen endurance limit
Se = endurance limit at the critical location of a machine part in the geometry and condition of use.

We can find out different factor like surface finish factor, size factor, loading factor, temperature factor, reliability factor, miscellaneous effects factor as per the guideline. (Joseph E. Shigley*et al*, Machine Engineering Design)

Now, we can find out endurance limit for all different materials.

Se = ka*kb*kc*kd*ke*kf*Se' = 75 MPa



N (No. of working cycles) = L_{min}X (100-Probability) / (100 X Miscellaneous Factor)

D (No. of working days) = N / (No. of working cycles per day)

Table 2 Number of Working Cycles and Number of Working Days with Probability

Sr. No.	Probability (%)	No. of working cycles for Silica Ramming Mass	No. of days working
1	100	231	25.7
2	90	236	26.2
3	80	240	26.7
4	70	245	27.2
5	60	249	27.7
6	50	254	28.2
7	40	259	28.7
8	30	263	29.3
9	20	268	29.8
10	10	273	30.3

We can understand from this table that all cases of induction furnace wall will work for minimum 231 working cycles and 25.7 days. The probability will be decreasing continuously with increase in life span and working days. It happens very rarely that induction furnace wall will work for 273 working cycles and 30.3 days so its probability is minimum 10% of cases can be having such a large life span.

Conclusion

Induction Melting Furnaces are highly used now- a-days for melting of different kinds of materials. The problem comes from the silica ramming mass of losing its material properties and failure occurs within 200-250 hours of lifetime. It will disturb production schedule as it requires time to replace the induction melting furnace wall of silica ramming mass. Explicit finite difference analysis is done for induction melting furnace refractory wall and validation is done with respect to experimental Results. Explicit finite difference analysis is done with respect to actual working conditions of induction furnace and material properties of silica ramming mass. Then S – log N Curves are plotted for Life Span Prediction.

We had found life span of different materials like silica ramming mass is 231-273 cycles. From the results of experimental study and explicit finite difference analysis of thermal fatigue failure of induction melting furnace wall, it can be seen that finite difference model exactly predicts the failure of the induction furnace refractory wall and the definite solution conditions in the finite difference numerical calculation are accurate. The fatigue life of the induction melting furnace refractory wall under thermal fatigue working conditions was predicted using stress-life method by plotting S – log N curves for silica ramming mass on the basis of explicit finite difference calculations and maximum thermal stress in the induction melting furnace refractory wall. We can use it as a linear to increase lifespan or we can use premixed silica ramming mass for economical and better working lifespan of induction melting furnace wall.

The accuracy of the fatigue life prediction for the induction melting furnace refractory wall depends upon temperature and thermal stress spectrum calculated at the critical point by explicit finite difference method and S-log N curves prepared from the material properties and boundary condition for silica ramming mass.

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