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Research Article

Numerical Study of Forced Convective Heat Transfer of Nanofluids inside a Vertical Tube

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Abstract

In this study, laminar forced convective heat transfer of nanofluids consisted of alumina/water and zirconia/water through a vertical tube under constant heat flux boundary condition was investigated numerically. Single phase and two phase mixture models were used for analyzing thermal behavior of nanofluids. Furthermore, effects of Reynolds number, nanoparticle types and nanoparticles volume fraction on the convective heat transfer coefficient were studied. The results of single phase and mixture models were compared with the experimental data. The results of the mixture model for prediction of the convective heat transfer coefficient showed better agreement with the experimental data, while the prediction of nanofluid mean bulk temperature distribution inside the tube by the single phase model was better than the mixture model compared to the experimental data. In addition, according to the results of numerical data, the convective heat transfer of nanofluids is higher than that of water similar to the experimental data. The average relative error for predicting convective heat transfer coefficient between experimental data and single-phase model was 13% and 8% for alumina/water and zirconia/water nanofluids, respectively while for mixture model was 8% and 5%.

Keywords: *Nanofluid, Forced convection, Heat transfer coefficient, Single phase model and Mixture model.*

1. Introduction

Fluids are used for heat transfer in many equipments. Metallic solids and oxides have larger thermal conductivity than the traditional fluids such as water, oil and ethylene glycol. Maxwell's (1873) investigation shows that if solid particles are added to the fluids, thermal conductivity of fluid-solid mixture is increased. Traditionally, solids with millimeter and micrometer particle sizes used to be applied, but these particles had some difficulties such as erosion, particle clogging and pressure drop in the tube because of their large sizes.
Modern technology makes it possible to have particles with nanometer dimensions. Because of their small sizes, problems such as pressure drop and particle clogging become insignificant for these fluids.

Masuda *et al*. (1993) reported an increase in thermal conductivity of liquid suspensions of Al_2O_3, SiO_2 and TiO_2 nanoparticles. Choi (1995) used the term nanofluid for the liquid suspension of nano-sized particles. Zhang *et al.* (2006) measured the thermal conductivities and thermal diffusivities of Al_2O_3/water, ZrO_2/water, TiO_2/water and CuO/water nanofluids and studied the effects of volume fraction, thermal conductivity of nanoparticles and temperature on thermal conductivities and thermal diffusivities of nanofluids. Their results demonstrate that the nanofluid thermal conductivities show no anomalous enhancement and can be predicted precisely by the Hamilton and Crosser model when the spherical nanoparticles are dispersed in fluids. Thermal conductivities of three nanofluids containing Al_2O_3, CuO and ZnO nanoparticles dispersed in water and ethylene glycol as the base fluids were experimentally determined and the effects of volume fraction and temperature on the thermal conductivities of these nanofluids were studied by Vajjha *et al*. (2009). Their results showed an increase in the thermal conductivity by increasing the volume fraction of nanoparticles and temperature and they developed a model for the thermal conductivity as a function of temperature, volume fraction, properties of nanoparticles and the base fluid, which had good agreement with the experimental data.

Some researchers have also proposed models for predicting the thermal conductivity of nanofluids. Maxwell (1904) proposed a model for predicting the thermal conductivity of two phase mixture consisting of continuous and discontinuous phases. His model was derived based on spherical shape for discontinuous phase. Hamilton and Crosser (1962) developed Maxwell's model to cover non-spherical particles. In addition, Murshed *et al* (2008) studied thermal conductivity and viscosity of nanofluids with spherical and cylindrical nanoparticles and proposed models for predicting thermal conductivity and viscosity of nanofluids.

Many researchers have also studied the convective heat transfer of nanofluids with different nanoparticles and

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base fluids under constant heat flux or constant wall temperature boundary conditions. The effects of particle volume fractions and Reynolds number on convective heat transfer coefficients and friction factor of Cu/water nanofluid for the laminar and turbulent flow under constant heat flux boundary condition were experimentally investigated by Li and Xuan (2002).

Zeinali *et al.* (2006, 2007) studied convective heat transfer for Cu/water, CuO/water and alumina/water nanofluids up to 3% volume concentrations for the laminar flow under constant temperature boundary condition. Hojjat *et al.* (2010) investigated experimentally laminar forced convective heat transfer of nanofluids flowing through a horizontal tube. Their results showed that all nanofluids had larger average and local convective heat transfer coefficients than that of the base fluid and shear-thinning behavior. Furthermore, He *et al.* (2007) studied convective heat transfer of TiO_2 nanoparticles in water as a base fluid in the laminar and turbulent flow regimes in a vertical tube. Effect of particle size, Reynolds number and volume concentration on the convective heat transfer coefficient were investigated and it was found that in a given Reynolds number and volume concentration, the convective heat transfer does not seem to be sensitive to the average particle size.

Laminar forced convection of a nanofluid consisting alumina and water was numerically studied by Izadi *et al.* (2009). They solved two dimensional elliptical governing equations to investigate the hydrodynamics and thermal behaviors of the fluid flow throughout an annulus. They used the single phase approach for nanofluid modeling. Bianco *et al.* (2009) used the single and two-phase model (discrete particles model) to investigate laminar forced convection flow of alumina/water nanofluid in a circular tube. Their investigation was accomplished for size particles equal to 100 nm. They reported a maximum difference in the average heat transfer coefficient between the single and two-phase models at about 11%. Lotfi *et al.* (2010) studied forced convective heat transfer of alumina/water nanofluid under constant wall flux boundary condition in a horizontal tube by the single phase, two phase Eulerian and two phase mixture models. The comparison between the numerical results and the experimental data shows that the mixture model is more precise than other models. In addition, Haghshenas Fard *et al.* (2010) investigated laminar forced convective heat transfer of three nanofluids under constant wall temperature boundary condition in a horizontal tube by the single phase and mixture models. They studied effects of nanoparticle sources, nanoparticle volume fraction and nanofluid Peclet number on heat transfer rate and compared the results of mixture model with those of single phase model. Two phase mixture model showed better agreement with experimental measurements. The results of their work showed that heat transfer coefficient is clearly increased by increasing particle concentration. The heat transfer enhancement is also observed by increase of Peclet number.

In this study, laminar forced convective heat transfer of alumina/water and zirconia/water nanofluids inside a vertical tube with constant heat flux boundary condition was investigated. zirconia and alumina nanoparticles were assumed to be spherical with the diameter of 50 nm. Single phase and mixture models were implemented for studying the thermal behavior of nanofluids. The results of numerical method compared with the experimental data (U. Rea *et al.*, 2009).

2. Governing equations and mathematical modeling

Figure 1 shows the tube with the diameter (D) of 4.5 mm and the length (L) of 1.01 m that was used in our study. The considering nanofluids were alumina/water and zirconia/water. Nanoparticles diameter was equal to 50 nm. The bulk densities of alumina and zirconia nanoparticles were 3920 and 5600 kg/m^3, respectively. Specific heat of alumina and zirconia were 880 and 418 J/kg.K, respectively. The nanofluid enters with a uniform axial velocity and the uniform temperature of T_{in}=295 K. On the tube wall, non-slip condition and constant heat transfer rate equal to 200 W were imposed (U. Rea *et al.*, 2009).

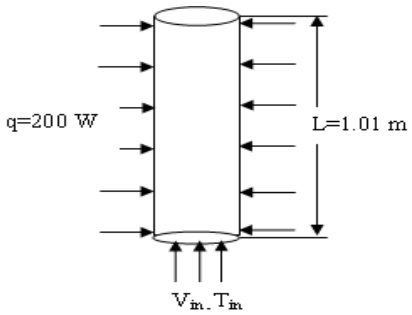

Figure 1. Geometrical configuration under study

2.1 Thermophysical properties of nanofluids

In the present study the thermophysical properties of zirconia/water and alumina/water nanofluids were calculated from the following equations:

$$\rho_{eff} = \varphi\rho_p + (1-\varphi)\rho_f \quad (1)$$

$$c_{p_{eff}} = \varphi c_{p_p} + (1-\varphi)c_{p_f} \quad (2)$$

Alumina-water nanofluid:

$$\frac{\lambda_{eff}}{\lambda_f} = 1 + 4.5503\varphi \quad (3)$$

$$\frac{\mu_{eff}}{\mu_f} = \exp(4.91\varphi/(0.2092-\varphi)) \quad (4)$$

Zirconia-water nanofluid:

$$\frac{\lambda_{eff}}{\lambda_f} = 1 + 2.4505\varphi - 29.867\varphi^2 \quad (5)$$

$$\frac{\mu_{eff}}{\mu_f} = 1 + 46.801\varphi + 550.82\varphi^2 \quad (6)$$

where φ was volume fraction of the nanoparticles, ρ, μ, λ and C_p are density, viscosity, thermal conductivity and specific heat, respectively. Subscripts eff, f and p indicate the effective, fluid and nanoparticles, respectively.

Equations (1) and (2) are based on the general correlations for classical two phase mixture (B.C. Pak *et al.*, 1998, S.E.B. Maiga *et al.*, 2004). Equation (3) was proposed to obtain the experimental data from curve fitting (U. Rea *et al.*, 2009, W.C. Williams *et al.*, 2008). Equation (4) was also obtained for calculation of thermal conductivity (U. Rea *et al.*, 2009, W.C. Williams *et al.*, 2008).

In this study, the single phase and two phase mixture models were used for analyzing the thermal behavior of nanofluid in the tube.

2.2 Single phase model

In the single phase model the nanofluid behaves as a conventional fluid but with the effective properties. The governing equations for steady state single phase model are expressed by the following equations:

Continuity:

$$\nabla.(\rho_m\vec{V}) = 0 \quad (7)$$

Momentum:

$$\nabla.(\rho_m\vec{V}\vec{V}) = -\nabla p + \nabla.(\tau - \tau_t) + \rho g \quad (8)$$

Energy:

$$\nabla.(\rho\vec{v}C_pT) = \nabla.(\lambda_{eff}\nabla T - C_p\rho_m\overrightarrow{vt}) \quad (9)$$

2.3 Mixture model

In the mixture model the nanofluid was considered as a single fluid with two phase approach that coupling between phases is strong. It was assumed that the two phases interpenetrated that each phase has its own velocity vector and within any control volume, there was a volume fraction of primary phase and also a volume fraction of the secondary phase. The governing equations for steady state single phase and mixture models are expressed by the following equations:

Continuity:

$$\nabla.(\rho_m\vec{V}_m) = 0 \quad (10)$$

Momentum:

$$\nabla.(\rho_m\vec{V}_m\vec{V}_m) = -\nabla p_m + \nabla.(\tau - \tau_t) + \rho_m g + \nabla.(\sum_{k=1}^{n}\varphi_k\rho_k\vec{V}_{dr,k}\vec{V}_{dr,k}) \quad (11)$$

Volume fraction:

$$\nabla.(\varphi_p\rho_p\vec{V}_m) = -\nabla.(\varphi_p\rho_p\vec{V}_{dr,p}) \quad (12)$$

Energy:

$$\nabla.(\sum_{k=1}^{n}\varphi_k\vec{V}_k(\rho_kH_k + p)) = \nabla.(\lambda_{eff}\nabla T - C_p\rho_m\overrightarrow{vt}) \quad (13)$$

where $\vec{V}_{dr,k}$ in the conservation of momentum equation is the drift velocity of secondary phase k, i.e. nanoparticles and defined as:

$$\vec{V}_{dr,k} = \vec{V}_k - \vec{V}_m \quad (14)$$

The shear stress is given by:

$$\tau = \mu_m\nabla\vec{V}_m \quad (15)$$

$$\tau_t = \sum_{k=1}^{n}\varphi_k\rho_k\overline{v_kv_k} \quad (16)$$

The slip velocity (relative velocity) is defined as the velocity of secondary phase (p) relative to the velocity of primary phase (f) :

$$\vec{V}_{pf} = \vec{V}_p - \vec{V}_f \quad (17)$$

The drift velocity is related to the relative velocity by the following equation:

$$\vec{V}_{dr,p} = \vec{V}_{pf} - \sum_{k=1}^{n}\frac{\varphi_k\rho_k}{\rho_m}\vec{V}_{fk} \quad (18)$$

The relative velocity is calculated from Eq. (17) proposed by Manninen *et al.* (1996) and Eq. (18) by Schiller and Naumann (1935) is used to determine the drag function f_{drag}:

$$\vec{V}_{pf} = \frac{\rho_p d_p^2}{18\mu_f f_{drag}}\frac{(\rho_p - \rho_m)}{\rho_p}a \quad (19)$$

$$f_{drag} = \begin{cases} 1 + 0.15Re_p^{0.687} & Re_p \le 1000 \\ 0.0183Re_p & Re_p > 1000 \end{cases} \quad (20)$$

The acceleration in Eq. (17) is:

$$a = g - (\vec{V}_m.\nabla)\vec{V}_m \quad (21)$$

2.4 Numerical method

This set of nonlinear differential equations was solved by control volume approach. Control volume technique converts the governing equations to a set of algebraic equations that can be solved numerically. For the convective and diffusive terms, a second order upwind method was used. Pressure and velocity were coupled using Semi Implicit Method for Pressure Linked Equations [SIMPLE] (S.V. Patankar, 1980).

A combination of different nodes was tested and finally a grid with 400 nodes for z-direction and 40 nodes for r-direction was considered for our work. In the vicinity of the tube wall and in the entrance region highly compacted grid points were used.

3. Results and discussions

In order to demonstrate the validity and also the precision of the results of the two phase mixture and single phase models, comparisons with the experimental data were made.

The heat transfer coefficient (h) and the Nusselt number (Nu) of flowing nanofluids are defined as follow:

$$h(x) = {q^{"}}/{(T_w(x) - T_m(x))} \quad (22)$$

$$Nu(x) = \frac{h(x).D}{\lambda_{eff}} \quad (22)$$

Figure 2 shows a comparison between local convective heat transfer coefficient for numerical results and experimental data (U. Rea *et al.*, 2009) versus Reynolds number at the axial position of z=0.3 m for water. The good agreement between the experimental data and the numerical results shows validation of the accuracy of the numerical model.

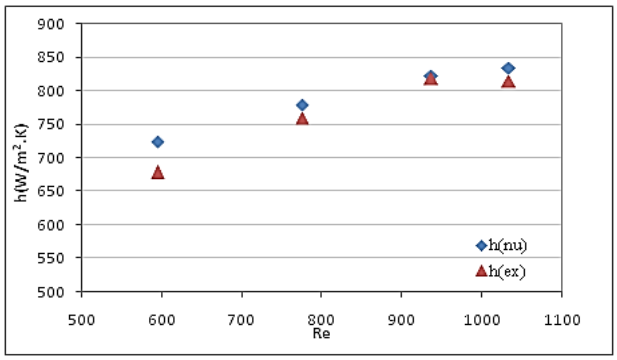

Figure 2. Local convective heat transfer coefficient of water versus Reynolds number at x=0.3 m.

Figures 3 and 4 depict the comparison of local convective heat transfer coefficient at x=0.16 m between the experimental data (U. Rea *et al.*, 2009) and the numerical results for %1.32 volume fraction of alumina/water and zirconia/water nanofluids versus Reynolds number respectively and it was observed that the results of mixture model were more precise than the single phase model. In addition, difference between the numerical results and the experimental data was decreased with increasing Reynolds number.

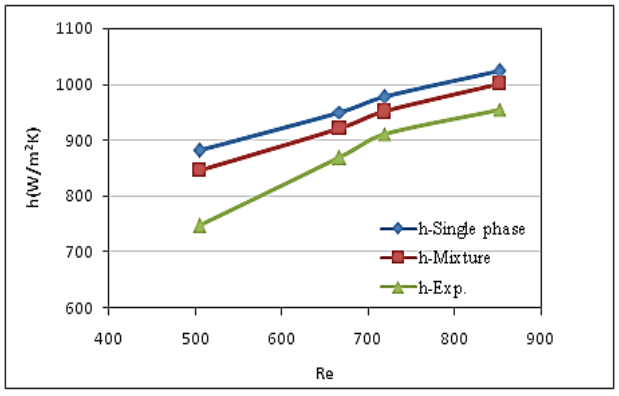

Figure 3. Local convective heat transfer coefficient of alumina/water nanofluid with 1.32 % vf versus Reynolds number at x=0.16 m.

Figure 5 displays the fluid mean bulk temperature versus axial position for alumina/water nanofluid with different volume concentrations for Re=850. This figure shows that the fluid mean bulk temperature is decreased for nanofluids by increasing the volume fraction of nanoparticles.

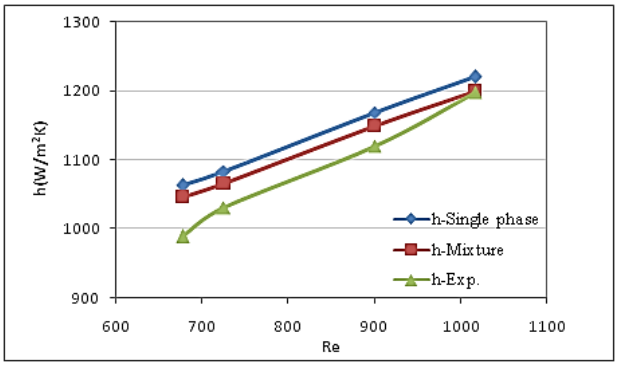

Figure 4. Local convective heat transfer coefficient of zirconia/water nanofluid with 1.32 % vf versus Reynolds number at x=0.16 m.

It was also observed that the fluid mean bulk temperature for φ=0.64% is lower with respect to the case of base fluid, and the two models predicted the mean bulk temperature decrease due to the presence of the nanoparticles. This behavior can be attributed to the improved thermophysical properties of nanofluid with respect to the base fluid. The product of specific heat and density is increasd for the nanofluid, therefore more energy is required to increase the fluid mean bulk temperature with respect to the pure water. It was also observed that as the volume concentration of nanoparticles is increased, the deviation between two models is increased and the prediction of single phase model for temperature is lower than that of mixture model.

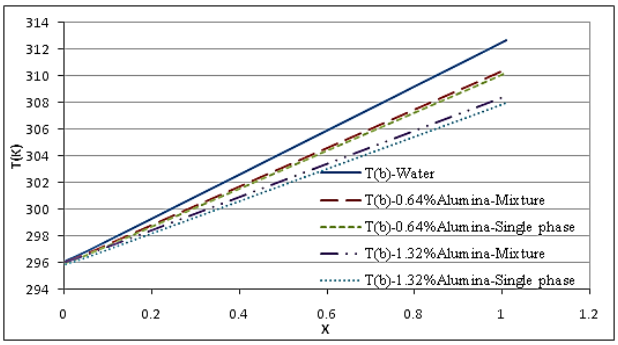

Figure 5. Fluid mean bulk temperature versus axial positions for water and alumina/water nanofluid at Re=850.

Figure 6 shows the wall temperature versus axial position for alumina/water nanofluid with different volume concentrations for Re=850 and it was found from this figure that a similar behavior to the mean bulk temperature was observed. As the figure shows, the deviation between the two models grows as the volume concentration of nanoparticles is increased, and the predicted temperature

values by the mixture model were higher than those obtained by the single phase model.

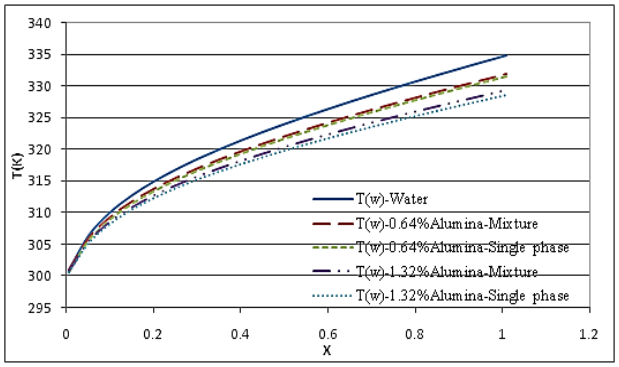

Figure 6. Wall temperature versus axial positions for water and alumina/water nanofluid at Re=850.

Figure 7 displays the predicted fluid mean bulk temperature by the single and mixture models versus the axial position compared to the experimental data for alumina/water nanofluid. It was observed that the single phase model had better prediction for the fluid mean bulk temperature distribution inside the tube for nanofluids.

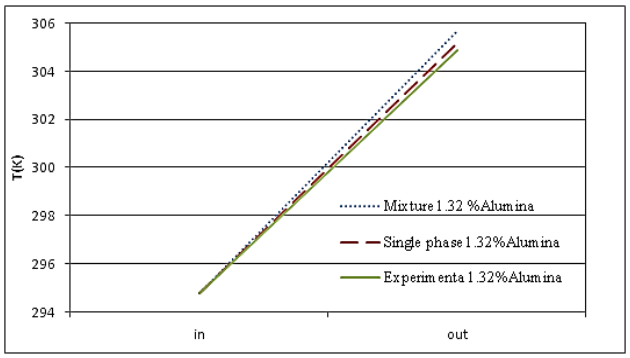

Figure 7. Mean bulk temperature versus axial positions for alumina/water nanofluid at Re=900.

Figure 8 shows the fluid mean bulk temperature and the wall temperature obtained by the single phase model versus axial position for alumina/water and zirconia/water nanofluids with 1.32 % volume concentration of nanoparticles. The fluid mean bulk and wall temperature for zirconia/water nanofluid was lower than that for alumina/water nanofluid. This can be explained by the higher product of specific heat and density for zirconia/water nanofluid compared to alumina/water nanofluid. Therefore more energy is required to increase the mean bulk temperature of zirconia/water nanofluid.

Figures 9 show Nusselt number versus Reynolds number at the axial position of x=0.3 m for water and alumina/water nanofluid with different volume fractions. The results show that adding 0.32, 0.64, 1.32 and 2.76 % volume concentration of nanoparticles to water increases Nusselt number about 0.5, 2.1, 5.4 and 15.3 for alumina/water nanofluid relative to pure water. The results

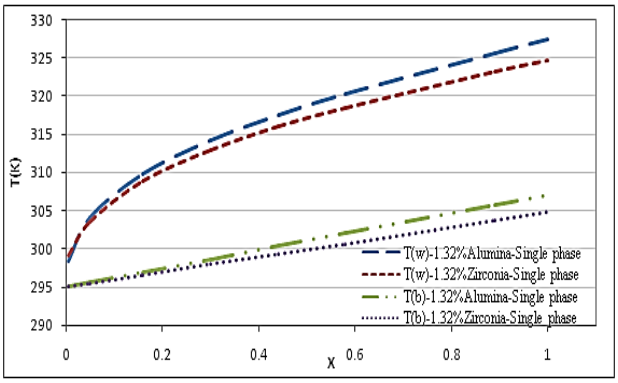

Figure 8. Wall temperature and fluid mean bulk temperature for nanofluids at Re=850.

also show that the relative differences between average Nusselt number of water and nanofluid inside the tube are increased as Reynolds number is increased. Larger Nusselt number for nanofluid compared to the pure water can be attributed to the improved thermophysical properties of nanofluid with respect to the base fluid.

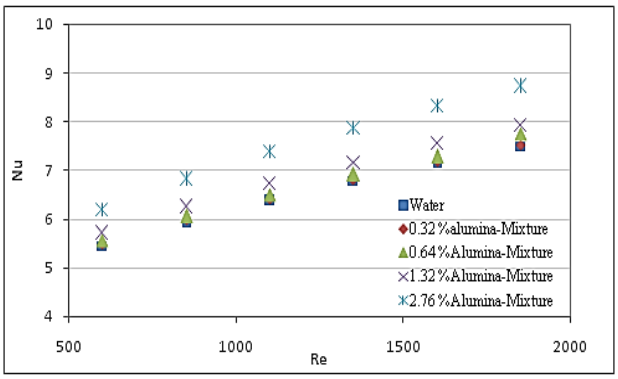

Figure 9. Local Nusselt number at x=0.3 versus Reynolds number for water and alumina/water nanofluid.

4. Conclusion

Laminar forced convective heat transfer of alumina/water and zirconia/water nanofluids inside a vertical tube under constant heat flux boundary condition was studied numerically. In order to demonstrate the validity and also precision of the models and the numerical procedure, comparison with the experimental data was made. Two different approaches, which were single phase model and mixture model, were studied. The results showed that the mixture model had better prediction for the convective heat transfer coefficient and the single phase had better prediction for fluid mean bulk temperature of nanofluids. In addition, it was found that nanofluid had better heat transfer than that of water.

Acknowledgement

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