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Research Article

Force Convection of Laminar Liquid Flow inside Pipe Exerted to Non-Uniform Heat Flux

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Abstract

In this study, the six cases of non-uniform heat flux are supplied on a circular pipe flow the liquid has Prandtl number is 13,400. The continuity, momentum, and energy equations were solved numerically by using finite volume technique are conducted with the commercial software FLUENT version (6.3) package program. The numerical liquid flow and force convection heat transfer inside pipe was performed with Reynolds number varying from (1 – 9) (in the steps of 2) plotted with value of Nusselt number. The results of temperature were compared with other previously published research that presented support the validity of results. Results have shown increase values of average Nusselt number with increasing of Reynold number at known Prandtl number, also maximum velocity in center of velocity profile increase with increasing of Reynolds number. The average Nusselt number increase with Reynolds number for each case then correlation its values.

Keywords: *force convection, non-uniform heat flux, laminar flow, Nusselt number.*

1. Introduction

Many applications in industrial forced a liquid through circular tube supplied on the surface multi-stage or non-uniform of heat flux such as plastics production in injection and extrusion molding, therefore heat transform to the fluid by force convection depending on heat flux as a function of position of pipe and properties of the fluid.

F. Kowsary, et al : (2007) presented analytical model for the pipe flow with constant heat flux in which leakage of the heat flux to the ambient is taken into account. Their results showed that the bulk temperature not rise linearly as in the constant heat flux. The temperature of the fluid and tube surface remains constant, while there is generated heat over the tube. The analytical solution yields a stationary length for the tube beyond which the change in the fluid temperature is insignificant.

C.P. Tso, et al : (2010) analyzed hydro-dynamically and thermally fully developed laminar heat transfer of non-Newtonian fluids between fixed parallel plates taking into account the effect of viscous dissipation of the flowing fluid. They considered that both the plates kept at different constant heat fluxes. They solved the energy equation, and in turn the Brinkman number and power-law index. Their results showed that heat transfer depends on the power-law index of the flowing fluid and viscous dissipation effects on heat transfer between parallel plates.

Chao-Kuang Chen, et al: (2005) studied the conjugate heat transfer problem of thermally developing, hydrodynamically developed turbulent flow in a circular pipe. They used inverse method to estimate the unknown heat flux on the external surface of the circular pipe based on temperature measurements taken at several different locations within the fluid. They applied the linear least-squares-error method to determine the unknown boundary conditions of the pipe flow. This method requires no prior knowledge of the functional form of the unknown wall heat flux, and yields solutions for the unknown conditions within a single computational iteration.

N. Luna, et al : (2002) studied conjugated heat transfer process for the thermal entrance region of a developed laminar forced convection flow of a power law fluid in a circular tube. They applied a known uniform heat flux at the external surface of the tube and solved the energy equation analytically using integral boundary layer approximation by neglecting the heat generation by viscous dissipation and the axial heat conduction in the fluid. Their results presented different parameters such as conduction parameter, the aspect ratio of the tube, and the index of power-law fluid.

Lutfullah Kuddusi, et al : (2007) studied the slip-flow in microchannels for all the eight thermal versions. They purposed to show the effect of different thermal boundary conditions on heat transfer in microchannel. The velocity and Temperature distribution, thus found are used to determine the average and Nusselt number. They found that rarefaction has a decreasing effect on heat transfer in

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the microchannels exposed to any of the eight thermal versions.

Chii-Dong Ho, et al : (2008) improved the heat transfer efficiency of concentric circular heat exchangers under uniform wall fluxes. They developed theoretically and the analytical solutions were achieved by using the orthogonal expansion technique with the Eigen-functions expanding in terms of an extended power series.

Their results showed that a considerable heat transfer efficiency improvement was obtained and the power consumption increment due to the device with external recycle.

J.A. Esfahani, et al: (2010) considered developing laminar pipe flow with non-uniform heat flux on the wall and determined generation of entropy due to both friction and heat transfer. They studied the effect of heat flux distribution on entropy generation. They suggested seven cases with the same rate of heat transfer at the wall but with various heat flux distribution. There the results showed that heat flux distribution affects the amount of entropy generation and it can be controlled by varying heat flux distribution or its rate of change.

Afshin J. Ghajar, et al : (1995) studied the boundary between forced and mixed convection in a horizontal circular straight tube with reentrant, square-edged, and bell-mouth inlets under uniform wall heat flux boundary condition. They represented the results by a given Reynolds number and the value of the parameter Gr×Pr. They correlated a horizontal pipe with three different inlets.

Elsayed A.M., et al : (2008) investigated force convection heat transfer characteristics of pulsating turbulent air flow in a pipe heated at uniform heat flux experimentally over a range of Reynolds number $10^4 < Re < 4\times10^4$ and frequency $6.6 \leq f \leq 68$ Hz. Their results showed that Nusselt number is strongly affected by both pulsation frequency and Reynolds numbers. They classified heat results according to turbulent bursting model.

L. Redjem-Saad, et al : (2007) investigated direct numerical simulations of heat transfer in a fully developed turbulent pipe flow with isoflux condition imposed at the wall are performed for a Reynolds number Re = 5500. The temperature fluctuations and turbulent heat fluxes are found to increase when increasing Prandtl number for $Pr \geq 0.2$. Probability density functions and joint probability density functions of velocity and temperature fluctuations are used to describe the characteristics of the turbulent flow and heat transfer.

Seo Yoon Chung, et al : (2003) performed a direct numerical simulation is performed for turbulent heat transfer in a concentric annulus at Re = 8900 and Pr = 0.71 for two radius ratios = 0.1 and 0.5 and wall heat flux ratio 1:0. Their numerical results showed that the turbulent thermal structures near the outer wall are more activated than those near the inner wall, which may be attributed to the different vortex regeneration processes between the inner and outer walls.

Bader Alazmi, et al : (2002) analyzed the effect of using different boundary conditions for the case of constant wall heat flux under Local Thermal Non-Equilibrium (LTNE) conditions. In addition to the above mentioned boundary conditions, five pertinent new boundary conditions are introduced. Pertinent parameters such as porosity, Reynolds number, Darcy number, inertia parameter, particle diameter, and solid-to-fluid conductivity ratio are considered to assess and compare the physical features of different boundary conditions under investigation.

Ammar F. Abdul Waheed: (2006) In this investigated laminar forced convection heat transfer of Newtonian and non-Newtonian fluids inside a duct of rectangular shape numerically for a wide range of the Reynolds numbers of (Re = 500, 1000, 5000 and 10000) with the Power law index (n) of power law model ranging from (0.1 to 2), and Prandtl number of (Pr =1,10, and 100). He suggested two types of boundary conditions. Stream function, vorticity formulated with finite difference to solve the continuity, momentum, and energy equations. The numerical results showed the peak of average Nusselt number occurs between $(0.1 \leq n \leq 2)$, depending upon Reynolds and Prandtl numbers. As the Reynolds number increases six different correlations to show the dependence of the average Nusselt number on the power law index, the Reynolds and Prandtl numbers.

Ali Ates, et al : (2010) investigated transient conjugated heat transfer in thick walled pipes for thermally developing laminar flow involving two-dimensional wall and axial fluid conduction. They solved the problem numerically by a finite-difference method for hydrodynamically developed flow in a two-regional pipe, initially isothermal in which the upstream region is insulated and the downstream region is subjected to a suddenly applied uniform heat flux. There the results gave by non-dimensional interfacial heat flux values, and it is observed that heat transfer characteristics are strongly dependent on the wall thickness ratio, wall-to-fluid thermal conductivity ratio, wall-to-fluid thermal diffusivity ratio and the Peclet number.

Huseyin Yapici, et al : (2004) analyzed the radial and axial heat conductions and thermal stresses in a pipe with uniform and non-uniform wall heat flux of fully developed laminar forced convective conjugate. They performed numerical calculations by using the FLUENT 4.5 and HEATING7 computer codes. The stress distribution has also been presented inside the pipe wall for all the cases. They gave ideas to help in designing and operating heat transfer devices.

Hussein A. Mohammed, et al: (2007) determined experimentally the effect of Reynolds number and the effect of the heat flux on laminar air flow under mixed convection heat transfer for hydrodynamically fully developed, thermally developing and thermally fully developed flow situations in a uniformly heated horizontal circular. They proposed to an empirical correlation for a horizontal cylinder. Their results showed that increase in the increase in the Nusselt number values as the heat flux increases and that free convection effects tended to decrease the heat transfer results at low Re while to increase the heat transfer results for high Re. The average Nusselt numbers were correlated with the (Rayleigh numbers/Reynolds numbers).

The present study is included using of six cases as shown in Fig. (1), each once is supplied on circular tube run laminar a incompressible steady state fluid flow comprise entrance region to fully developed flow. The velocity profile in additional to local and average Nusselt numbers are estimated in many figures in each cases, then limit suitable case that give high average Nusselt number.

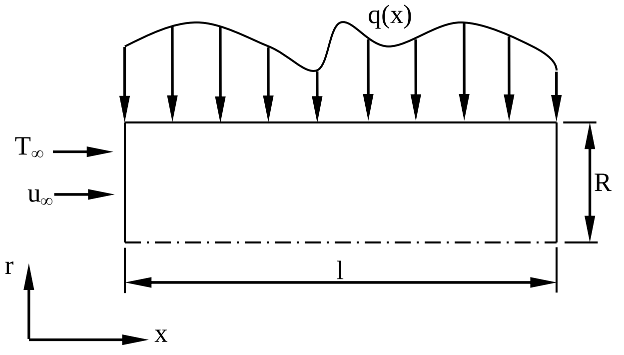


Fig. 1. Geometry and coordinate system

2. Governing Equations

The flow of fluid is governed by continuity, momentum (Navier-Stokes), and energy equations for steady, constant properties, two dimensional, incompressible, and Newtonian laminar flow also, viscous dissipation function is neglected to the special case of axisymmetric flow, where there are no circumferential variations. These equations mathematically form partial differential shapes in cylindrical coordinates are(Joseph A. Schetz et al,1996; Warren M et al,1998):

1. Continuity eq.

$$\frac{1}{r}\frac{\partial}{\partial r}(r.v)+\frac{\partial u}{\partial x}=0 \tag{1}$$

2. Momentum eq.

in r-radial direction:

$$\rho\left(v\frac{\partial v}{\partial r}+u\frac{\partial v}{\partial x}\right)=-\frac{\partial P}{\partial r}+\mu\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r.v)\right]+\frac{\partial^2 v}{\partial x^2}\right\} \tag{2}$$

In x-axial direction:

$$\rho\left(v\frac{\partial u}{\partial r}+u\frac{\partial u}{\partial x}\right)=-\frac{\partial P}{\partial x}+\mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u}{\partial r}\right]+\frac{\partial^2 u}{\partial x^2}\right\} \tag{3}$$

3. Energy eq.

$$\rho.c_p\left(v\frac{\partial T}{\partial r}+u\frac{\partial T}{\partial x}\right)=k\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right]+\frac{\partial^2 T}{\partial x^2}\right\} \tag{4}$$

above equations are solved by FLUENT program are based on the finite volume method, in which fluid region of pipe flow is discretized into a finite set of control volumes (mesh).

Some non-dimensional parameters of hydrodynamic and heat transfer characteristics are needed in the calculations then correlation that plays the important role to determine analysis type:

$$\mathrm{Re}=\frac{\rho\times u_\infty\times D}{\mu} \tag{5}$$

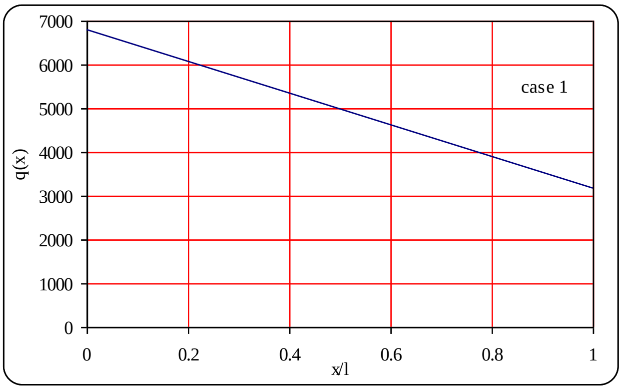
$$\mathrm{Pr}=\frac{\rho\times c_p}{k} \tag{6}$$

$$Nu=\frac{h\times D}{k} \tag{7}$$

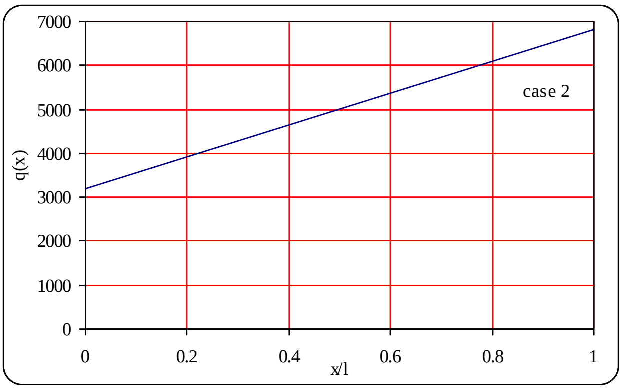
$$\overline{Nu}=\frac{1}{l}\int_0^l Nu.dx \tag{8}$$

3. Geometry and Boundary Conditions

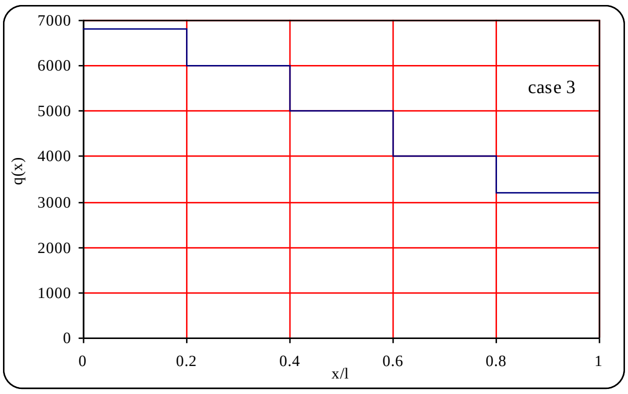
Cylindrical shape of the pipe has l/D = 40, shown in Fig. 1. horizontal direction of no-slip flow and the bottom wall represented center line of the pipe, while the top is the wall was supplied with non-uniform heat flux and the velocity of the flow near the wall would be equal to zero, while it's at center is maximum value. The temperature and velocity of the flow is (u_∞, T_∞) are uniform values of entrance region or inlet boundary conditions.



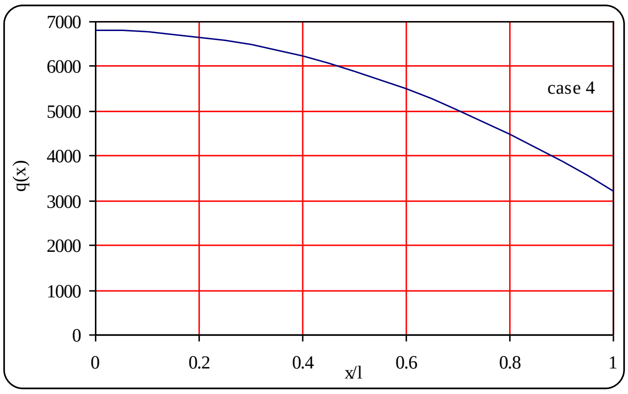
Case 1



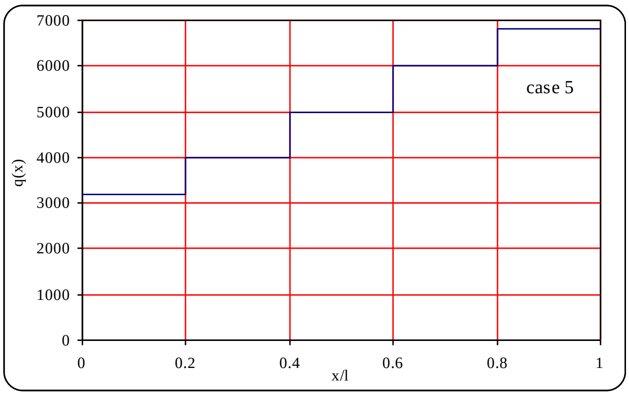
Case 2



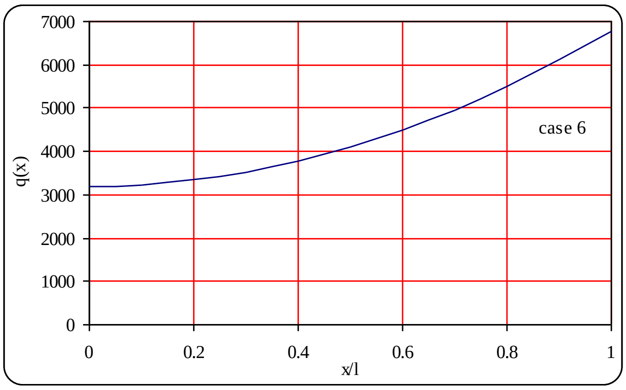
Case 3



Case 4



Case 5



Case 6

Fig. 2. six cases of non-uniform heat flux at wall of the circular pipe.

Six cases of non-uniform heat flux in (W/m^2) would supply on the top wall of the pipe in perform of the model as shown in Fig. 2. can be represented by algebraic and logic equations are:

1. case 1: $q(x) = 6800 - 3600x$ (first order negative slope).
2. case 2: $q(x) = 3200 + 3600x$ (first order positive slope).
3. case 3: $q(x) = 6800$ for $0 \le x \le 0.2$ (steps down value).
   $= 6000$ for $0.2 \le x \le 0.4$
   $= 5000$ for $0.4 \le x \le 0.6$
   $= 4000$ for $0.6 \le x \le 0.8$
   $= 3200$ for $0.8 \le x \le 1$
4. case 4: $q(x) = 6800 - 3600x^2$ (second order negative slope).
5. case 5: $q(x) = 3200$ for $0 \le x \le 0.2$ (steps up value).
   $= 4000$ for $0.2 \le x \le 0.4$
   $= 5000$ for $0.4 \le x \le 0.6$
   $= 6000$ for $0.6 \le x \le 0.8$
   $= 6800$ for $0.8 \le x \le 1$
6. case 6: $(x) = 3200 + 3600x^2$ (second order positive slope).

4. Numerical Simulation

The governing equation (continuity, momentum, and energy) are solved numerically through the computational fluid dynamics package FLUENT with (UDF) user define function that allows to define wall heat flux as a function of position along pipe length, in which all cases have special profile different of one to other. This function can be added by written in C++ code language that will be called by FLUENT program to specify the boundary condition. This program conducted finite volume method to integrate the governing equations. They solve it in the discretized sequentially.

The problem grid (mesh) is generated by using a commercial grid generator GAMBIT, in watch FLUENT using this mesh to define the model, node, and boundary conditions. The grid structure consists of uniform quadrilateral or rectangular shape has (4800) cells and (4990) nodes for 160 × 30 mesh size.

5. Results and Discussion

The results of computational fluid dynamics and heat transfer characteristics have been estimated by FLUENT program at residual error is 1×10^{-6} at each perform base on the Reynolds number are (1, 3, 5, 7, 9). Local and average Nusselt are determined at each case and Reynolds numbers. Finally correlate maximum velocity and average Nusselt number with Reynolds number of each one of the six type cases.

The results has been validated with other identified result of Ref. (J.A. Esfahani et al,2010) the temperature profiles at five longitudinal locations x = (0.1, 0.3, 0.5, 0.7, and 0.9 m) along the pipe for cases (3 & 5) as shown in Figs. 3, 4 therefore it's gave a good agreement for five location in dimensionless radial direction to compromise wide area of the fluid medium in two perpendicular direction of the pipe.

The velocity profiles are represented with Reynolds number in Fig. 5. Note the maximum velocity at center line of the circular pipe at known Reynolds number, while start velocity decreasing in direction the wall of the pipe then become zero near wall the pipe, also maximum velocity is increase with increasing with Reynolds number attributed to theory of boundary layer in internal flow. After plot the relationship between ratio of maximum to inlet velocity with Reynolds number in Fig. 6 shown that it's increase approximate linearly then correlation of this relationship by equation is:

$$\frac{u_m}{u_\infty} = 3.575 \times \mathrm{Re} \quad (9)$$

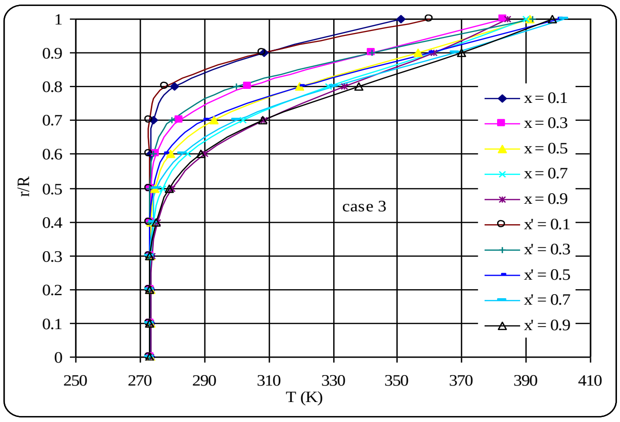


Fig. 3. Comparison between Ref. (J.A. Esfahani et al,2010) denoted (x') and present study denoted (x) of Radial temperature profiles at longitudinal locations along the pipe for case 3.

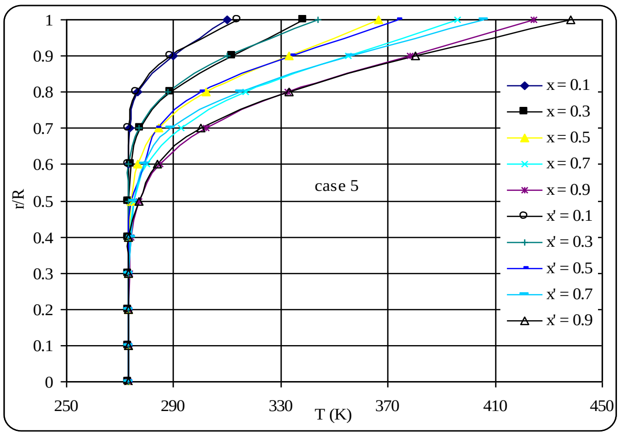


Fig. 4. Comparison between Ref.(J.A. Esfahani et al,2010) denoted (x') and present study denoted (x) of Radial temperature profiles at longitudinal locations along the pipe for case 5.

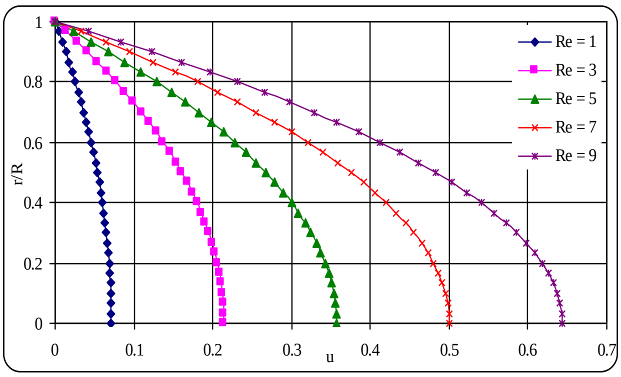


Fig. 5. Velocity profile based on Reynolds No.

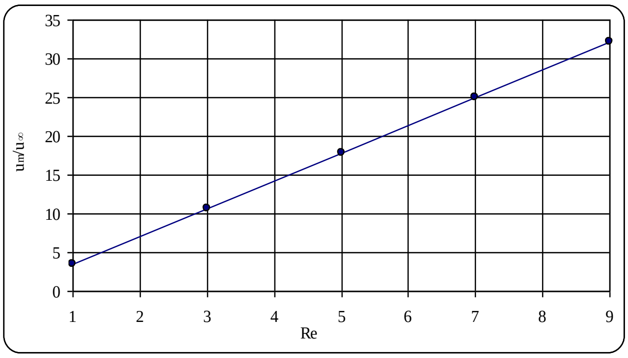


Fig. 6. Maximum to inlet velocity ratio with Reynolds number

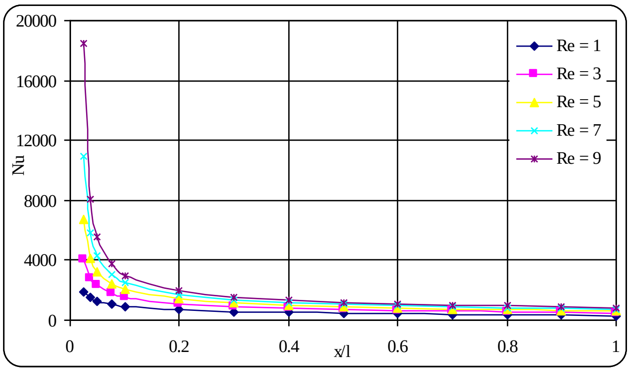


Fig. 7. Local Nusselt number vs. non-dimensional axial direction for case 1.

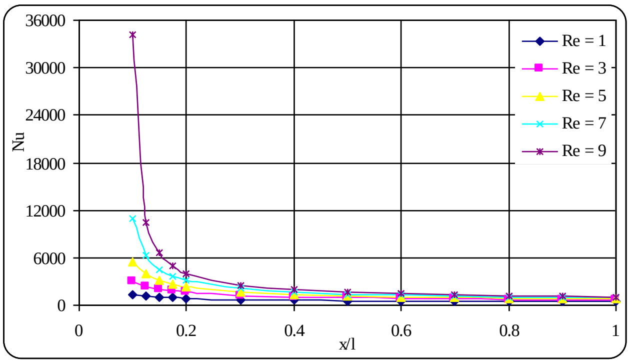


Fig. 8. Local Nusselt number vs. non-dimensional axial direction for case 2.

The Figs. 7, 8, 9, 10, 11, and 12 are represented the local Nusselt number along axial direction of the pipe. Note in six above graphs the local Nusselt number decrease along dimensionless axial direction for Re = 1, 3, 5, and 7 but in Re = 9 suddenly at start is high drop in it's value after 1/5 of the pipe length start it's value decrease but very slowly to end section of the pipe due to the changes of flow velocity near the inlet (entrance region) and conception of boundary layer at constant value of Prandtl number.

Also the Reynolds number increases, local Nusselt number increase for each case of non-uniform heat flux attributed to the disturbance of boundary layer, that is when Reynolds number increase both thermal and hydrodynamic boundary layers become thinner, therefore further heat is transferred from the wall that exerted to heat flux to the liquid.

To comparison between six cases to determine better case that can be obtain high rate of heat transfer with range of Reynolds number is (1, 3, 5, 7, and 9), therefore need to specify the average Nusselt number in each case as shown Fig. 13. Observe that cases 5 & 6 have same values approximately of curve but at Re = 7.5 – 9, curve of case 5 have values higher that it's case 6. After these two cases, note case 2 locate between cases 5 & 6 and other cases 1, 2, 4 that may be comprise one band, in which these cases 1, 3, and 4 approximately convergent but when work zoom obtain increasing arrangement is case 1, 4, and 3 respectively.

After plotted relationship between average Nusselt numbers with Reynolds number, the average values were correlated for each cases as exponent function in form $\overline{Nu} = C\,\mathrm{Re}^{n}$ to can be easy in calculations as:

Case 1: $\overline{Nu} = 457.01 \times \mathrm{Re}^{0.4898}$ (10)

Case 2: $\overline{Nu} = 570.52 \times \mathrm{Re}^{0.6425}$ (11)

Case 3: $\overline{Nu} = 445.77 \times \mathrm{Re}^{0.4751}$ (12)

Case 4: $\overline{Nu} = 450.28 \times \mathrm{Re}^{0.4747}$ (13)

Case 5: $\overline{Nu} = 575.02 \times \mathrm{Re}^{0.7225}$ (14)

Case 6: $\overline{Nu} = 585.38 \times \mathrm{Re}^{0.7081}$ (15)

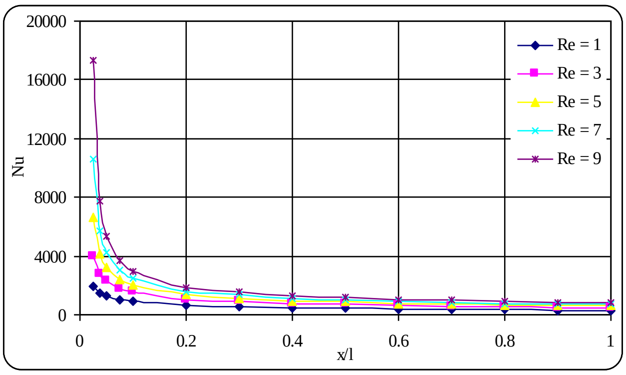


Fig. 9. Local Nusselt number vs. non-dimensional axial direction for case 3.

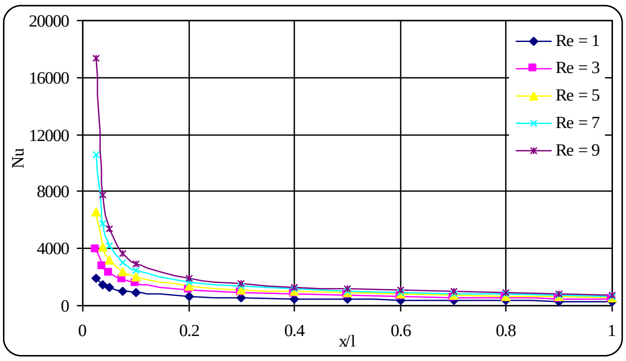


Fig. 10. Local Nusselt number vs. non-dimensional axial direction for case 4.

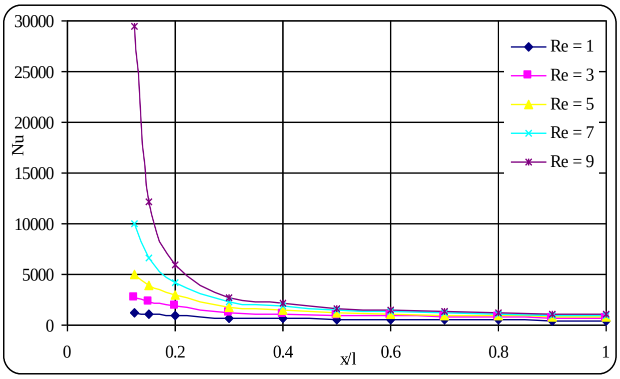


Fig. 11. Local Nusselt number vs. non-dimensional axial direction for case 5.

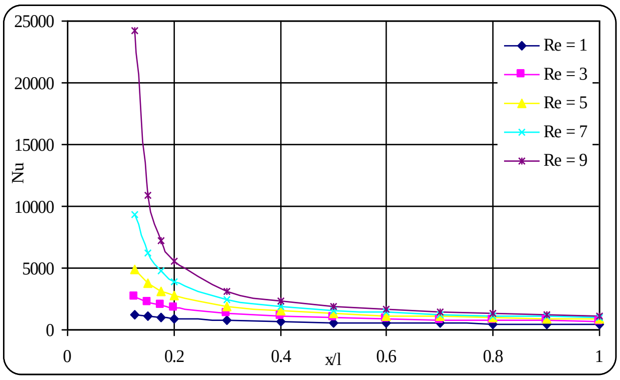


Fig. 12. Local Nusselt number vs. non-dimensional axial direction for case 6.

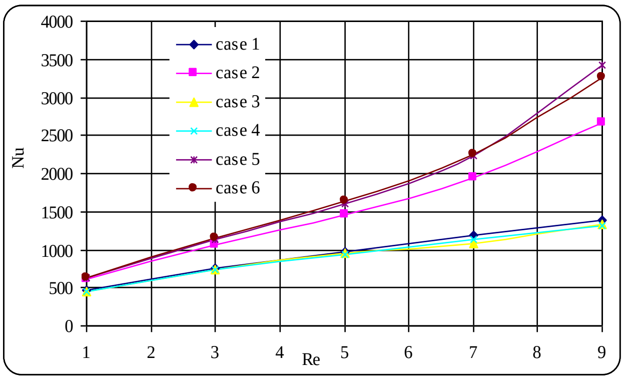


Fig. 13. Average Nusselt number vs. Reynolds number.

6. Conclusions

1. when Reynolds number increase, maximum to inlet uniform velocity ratio is increase and they have linear relationship as in Eq. 9.
2. local Nusselt number drop suddenly in entrance region then decrease gradually in fully developed flow with dimensionless axial direction for Re = 9 in each case, but another values of Re = (1, 3, 5, 7) local Nusselt no. decrease slowly in full range of dimensionless axial direction.

3. for all six cases, local Nusselt number increase with Reynolds number according to its values that take in this work (Re = 1 to 9, step 2).
4. average Nusselt number increase with increase Reynolds number for each considered cases.
5. optimum case is that have high average Nusselt number, after compare between its value of six case by Fig. 13 show that case 5 is suitable to give high rate of convection heat transfer.
6. finally, the average Nusselt number with Reynolds number relationship were correlated in Eqs. 10, 11, 12, 13, 14, 15.

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