Research Article

Lattice Boltzmann Analysis of 2-D Natural Convection Flow and Heat Transfer within Square Enclosure including an Isothermal Hot Block

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Abstract

In the present investigation, physics of natural convection flow and heat transfer in closed enclosure including an isothermal heat block are analyzed by double population approach of lattice Boltzmann method (LBM). The sidewalls are isothermally cooled at a constant temperature while the upper and bottom walls are considered to be adiabatic except for the rectangular block heated at a uniform temperature. The simulations were performed for a Prandtl number fixed to 0.71. Main attention has been focused on the effects of Rayleigh number ($10^3 \leq Ra \leq 10^6$), position ($0.1 \leq x_c \leq 0.4$), length ($0.1 \leq a \leq 0.6$) and height ($0 \leq b \leq 0.4$) of the active block upon dynamic and thermal fields of fluid. It is found that the present approach of thermal LB model D2Q4-D2Q9 produces similar results by comparison with former predictions. Besides, the computational results show that the parameters governing the problem have a considerable effects on the flow patterns and the temperature repartition and consequently on the heat transfer rate. By increasing Rayleigh number, average Nusselt number along upper and lateral surfaces of the heater increases causing an enhancement of convective mode in enclosure. A weakening of heat transfer rate is obtained with increasing position, length as well as height of hot source.

Keywords: Thermal lattice Boltzmann model, Natural convection, Square enclosure, active block, Heat transfer.

Introduction

In the last decades, lattice Boltzmann method (LBM) becomes a powerful and efficient method over conventional techniques to solve various problems of natural convection in 2-D confined/semi-confined medium encountered in many areas of sciences and engineering. Besides, implementation of boundary conditions with double populations of LB approach is easy and convection operator is linear. Convective heat transfer has lots of applications in nature and industry, such as, cooling of electronic devices, solar energy systems, building/tunnel engineering. Owing to this wide range of applications, so far several researchers focused their attention on natural convection in enclosures with localized square or circular heated source. For example, A.Ortega performed experimental investigation on natural convection air cooling of a discrete source on a conducting board in a shallow horizontal enclosure. Results show that the heat transfer coefficients have a distinct behavior at high aspect ratio in which the dominant length scales were related to the source. Natural convection in a vertical square enclosure containing heat generating conducting body is

studied numerically by J. Oh et al.. They exhibited that increasing Rayleigh number enhances the heat transfer. D.G.Roychowdhury performed numerical simulation of two dimensional natural convective flow and heat transfer around a heated cylinder kept in a square enclosure by finite volume technique. It is observed that the patterns of recirculation flow and thermal stratification in the fluid are significantly modified with aspect ratio between enclosure and cylinder dimensions. E.Braga analyzed steady laminar natural convection within a square cavity filled with a fixed volume of conducting solid material consisting of either circular or square obstacles. He observed that the average Nusselt number for cylindrical rods was slightly lower than those for square rods. Natural convection around a tilted heated square cylinder kept in an enclosure with three aspects ratio has been studied by Arnab et al.. It is found that flow pattern, thermal stratification and overall heat transfer are modified versus aspect ratio. Jami et al. have analyzed a numerical investigation on laminar convective flows in a differentially heated square enclosure with a heat-conducting cylinder at its center. They concluded that for a constant Rayleigh number, the average Nusselt number at the hot and cold walls varied linearly with temperature-difference ratio. Kim et al. have

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considered the problem of natural convection in a square enclosure with a circular cylinder at different vertical locations. They found that the size, number and formation of the cell strongly depend on the Rayleigh number and the position of the inner heated circular cylinder. N.A.Che Sidik investigated the fluid flow behavior and heat transfer mechanism from a heated square cylinder located at various heights inside an enclosure. Computational results demonstrated that the flow pattern, number, size and formation of vortices and heat transfer mechanism are critically dependence on Rayleigh number and the position of heated inner square cylinder in enclosure. M.Paroncini analyzed the influence of the height of hot source on natural convective heat transfer in a square cavity. He exhibited that the average Nusselt number increases with the Rayleigh number for different source heights. Natural convection in a concentric annulus between cold outer square cylinder and a hot inner circular cylinder is simulated by N.A.Che Sidik. The computations show that the Rayleigh number greatly affects the flow pattern and temperature distribution inside the enclosure.

Lee et al. have carried out a numerical analysis on natural convection induced by a temperature difference between a cold outer square cylinder and a hot inner circular cylinder. They investigate the effects of the inner cylinder location in an enclosure and the buoyancy-induced convection on heat transfer and fluid flow. Hussain et al. have investigated 2-D natural convection with a uniform heat source applied on the inner circular cylinder in a square air filled enclosure. Numerical solutions yield a two cellular flow field between the inner cylinder and the enclosure and the average Nusselt number behaves nonlinearly as a function of locations. S.H. Hussain study free convection heat transfer in a differentially heated square enclosure filled by air with an interior heat generating conducting solid circular cylinder at different diagonal locations. It is found that the average Nusselt number at the cold right side wall increases for all values of location of interior cylinder along the enclosure diagonal when the heat generation value and Rayleigh number increases. A finite element method is used by S.Parvin to simulate laminar and MHD free convection flow in a cavity with a heated body. He concluded that Rayleigh number and diameter of the body are a significant effect on the flow and temperature fields. J.Guiet et al have conducted an investigation of natural convection of nanofluids in a square enclosure with a protruding heater. They concluded that the heat transfer is improved vs. size and position of the heating block in enclosure. A numerical study is presented by X.Zhang to unveil the effect of location ratio variation of cold isolated plate and hot isolated vertical plate on the natural convection in an enclosure. The simulations show that the average Nusselt number tends to decrease with the increase of vertical location ratio for hot plate. J.Guiet investigated a numerical study on natural convection from a protruding heater located at the bottom of a square cavity filled with a copper-water nanofluid. For an isothermal heater, the heat transfer is improved when the size of the heating source increases. H.Khozeymehnez had carried

out a comparison of natural convection around a circular cylinder with a square cylinder inside a square enclosure. At low Rayleigh numbers and for both cylinders, the bifurcation from the bi-cellular vortices to uni-cellular vortex occurs when an inner cylinder is placed at a certain distance from the center of the enclosure. Recently, heat transfer and fluid flow due to natural convection in air around heated square cylinders of different sizes inside an enclosure are investigated by R.K.Singh. It is proved that Nusselt number dependent of temperature and heated square cylinder can absorb more energy by increasing heater size.

The relevant literature mentioned above has observed mainly the effects of heater location in confined medium, aspect ratio and thermal boundary condition. However, there is little information about the effects of Rayleigh number, position, length and height of hot block on the natural convection in enclosure. In the present computational investigation by thermal lattice Boltzmann model, the main objective is to analyze the effects of pertinent parameters in the following ranges: Rayleigh number Ra=10^3-10^6, position x_c=0.1-0.4, length a=0.1-0.6 and height b=0-0.4 of an isothermal rectangular block located on the bottom wall of a square enclosure on fluid flow and heat transfer of natural convection.

2. Description of the physical problem

Fig.1 illustrates the physical problem of interest with boundary conditions. The square enclosure of length (L) is heated from below with localized hot rectangular block. The heater of length (a) and height (b) is maintained at a constant temperature T_h. The vertical walls are isothermally cooled at a constant temperature T_c and the horizontal walls are considered to be insulated except the active source.
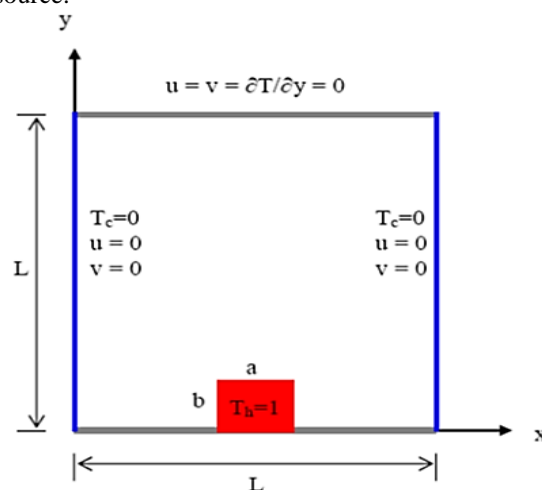

Fig.1 Schematic geometry of physical problem

In the present investigation, we employed double population thermal lattice Boltzmann model in order to provide a fundamental understanding of the effects of Rayleigh number (Ra) ranging from 10^3 to 10^6, source position (x_c) from 0.1 to 0.4, source length from 0.1 to 0.6 and source height from 0 to 0.4, on natural convection

flow around a hot block in enclosure. Dimensionless parameters used in the present thermal LB model are: a=l/L, b= h/L, x_c=x/L, Rayleigh number ($Ra = \dfrac{g \beta \Delta T L^3}{\chi \nu}$),

Prandtl number (Pr=$\dfrac{\nu}{\chi}$) and average Nusselt number at each hot wall of heater calculated by a second order finite difference ($Nu_w = \dfrac{1}{\varepsilon} \sum \dfrac{3\theta_0 - 4\theta_1 + \theta_2}{2}$).

The reference quantities used in the present investigation are presented as: L_0=L, U_0=χ/L, t_0=L²/ χ, p_0=$\rho_0 U_0^2$ and ΔT=T_h-T_c used for length, velocity, time, pressure and relative temperature, respectively.

Next, solutions were assumed to converge when the following convergence criterion was satisfied for all dependent variables at each point in the computation domain:

$$\left| \frac{\Phi(t+5000) - \Phi(t)}{\Phi(t)} \right| \leq 10^{-4} \qquad (1)$$

where Φ stands for a dependent variables θ , u and v.

In the present work, to investigate natural convection problems with lattice Boltzmann approach, we used the characteristic velocity defined by $U = \sqrt{g\beta\Delta TL}$ as a reference scale to check the compressibility limit and for the sake of comparison with previous studies.

3. Lattice Boltzmann method (LBM) and boundary conditions

For the present incompressible thermal problem, two standard models D2Q9 and D2Q4 of LBM are used, for the flow and temperature fields, respectively (Fig.2). The computation approach of thermal lattice Boltzmann model is based on two distribution functions originally proposed by He et al.. After introducing Bhatnagar-Gross-Krook approximation (BGK), double population functions of lattice Boltzmann equation with external force can be written as:
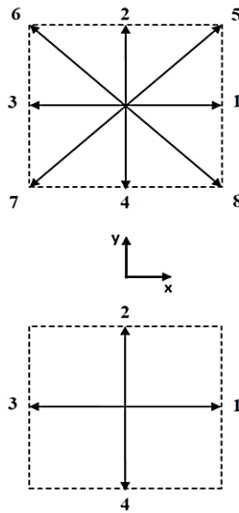

Fig.2 Lattice arrangements of models, D2Q9 (top) and D2Q4 (bottom)

For the flow field:

$$f_i(\vec{x}+\vec{\xi_i}\delta t, t+\delta t) - f_i(\vec{x},t) = -\frac{1}{\tau_f}[f_i(\vec{x},t) - f_i^{eq}(\vec{x},t)] + \delta t F_i \quad (2)$$

For the temperature field:

$$g_i(\vec{x}+\vec{\xi_i}\delta t, t+\delta t) - g_i(\vec{x},t) = -\frac{1}{\tau_g}[g_i(\vec{x},t) - g_i^{eq}(\vec{x},t)] \quad (3)$$

where f_i and g_i are density distribution functions, τ_f and τ_g characterize the single relaxation times resulting from the BGK approximation for the collision operator, F_i is the external force in direction of lattice velocities $\vec{\xi_i}$ and the equilibrium density distribution functions can be formulated as

$$f_i^{eq} = w_i \rho [1 + 3\frac{\vec{\xi_i}.\vec{u}}{c^2} + \frac{9}{2}\frac{(\vec{\xi_i}.\vec{u})^2}{c^4} - \frac{3}{2}\frac{\vec{u}^2}{c^4}] \qquad (4)$$

$$g_i^{eq} = w_i^{'} \theta [1 + 3\frac{\vec{\xi_i}.\vec{u}}{c^2}] \qquad (5)$$

The weighting factors w_i for D2Q9 model are given as $w_0 = \dfrac{4}{9}$, $w_{1-4} = \dfrac{1}{9}$, $w_{5-8} = \dfrac{1}{36}$ and $w_i^{'} = 0.25$ for D2Q4 model.

The discrete velocities $\vec{\xi_i}$ are defined as: $\vec{\xi_0} = (0,0)$, $\vec{\xi_{1-4}} = (\pm c, 0)$ and $\vec{\xi_{5-8}} = (\pm c, \pm c)$.

$c = \dfrac{\Delta x}{\Delta t}$, Δx and Δt are the lattice space and the lattice time step size, respectively, which are set to unity. After the compute of distribution functions, the macroscopic variables such as the flow density ρ , flow velocity $\vec{u} = (u,v)$ and temperature θ are defined as:

$$\rho = \sum_{i=0}^{8} f_i \qquad (6)$$

$$\rho\vec{u} = \sum_{i=0}^{8} \vec{\xi_i} f_i \qquad (7)$$

$$\theta = \sum_{i=1}^{4} g_i \qquad (8)$$

The governing equations of natural convection problem; continuity, momentum and energy can be recovered through the Chapman-Enskog expansion (He et al.) under incompressible limit assumption $Ma = |\vec{u}|/c_s \ll 1$ and without external force term, are as follows:

$$\begin{cases} \nabla.\vec{u} = 0 \\ \partial_t \vec{u} + \nabla.(\vec{u}\,\vec{u}) = -(\nabla p)/\rho + \nu\nabla^2\vec{u} \\ \partial_t \theta + \nabla.(\vec{u}\,\theta) = \chi\nabla^2\theta \end{cases} \qquad (9)$$

where $p = \rho c_s^2$ is the pressure from the equation of the state for the ideal gas, $c_s = \dfrac{c}{\sqrt{3}}$ is the sound speed. The time relaxations in mesoscopic world are related to the kinetic viscosity and thermal diffusivity in the macroscopic world as follow:

$$\tau_f = 3\nu + 0.5 \qquad (10)$$

$$\tau_g = 2\chi + 0.5 \qquad (11)$$

In the present approach of LBM, Boussinesq approximation is applied to modeling external force term. All fluid properties are assumed to be constant except the fluid density given by $\rho = \rho_0(1-\beta(T-T_r))$, where ρ_0 is a reference fluid density, then the external buoyant force $\rho_0\vec{G} = -\rho_0\beta(T-T_r)\vec{g}$ appearing in momentum equation will be expressed as

$$F_i = \frac{\vec{G}.(\vec{\xi_i}-\vec{u})}{c_s^2} f_i^{eq} \qquad (12)$$

Following these considerations, $\left|\vec{u}\right| << \xi_i$, $f_i^{eq} \approx w_i\rho(x,t)$ and $T_r = 0$, external body force is defined by the following expression:

$$F_i = -3w_i\rho(\vec{x},t)\beta T(\vec{x},t)\vec{g}.\vec{\xi_i} \qquad (13)$$

The boundary conditions on all solid walls associated to the problem are defined by bounce-back boundary conditions, which mean that incoming boundary populations are equal to out-going populations after collision. In LBM, distribution functions out of the computation domain are known from the streaming process. The unknown distribution functions, as dotted lines, are those towards the domain (Fig. 3). For instance, unknown distribution functions of the flow filed in the west boundary of the enclosure are determined by the following conditions:

$$f_{1,0} = f_{3,0}$$
$$f_{8,0} = f_{6,0} \qquad (14)$$
$$f_{5,0} = f_{7,0}$$

For the thermal boundary conditions, distribution function at upper wall of hot block is evaluated by the following expressions:

$$g_2 = 0.5 - g_4 \qquad (15)$$

The adiabatic boundary condition is transferred to Dirichlet-type condition using the conventional second-order finite difference approximation as:
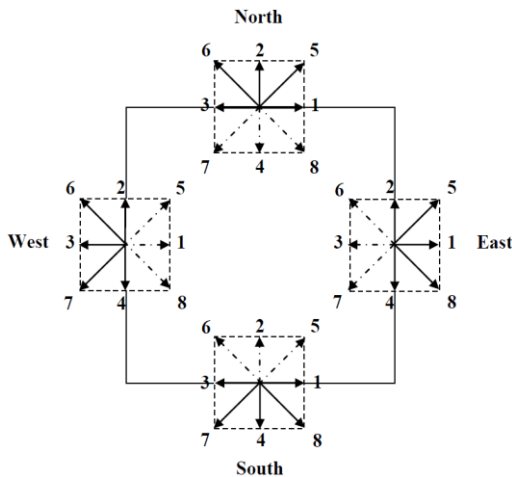
$$g_{wall} = (4g_1 - g_2)/3 \qquad (16)$$

Fig.3 Known and unknown distribution functions at the domain boundaries

4. Results and discussion

In the present investigation, standard models D2Q9-D2Q4 of lattice Boltzmann method (LBM) are used to unveil the effect of heat rectangular block over natural convection in a square enclosure filled by air with fixed Prandtl number at 0.71. Main interest is focused on the effects of pertinent parameters; Rayleigh number ($10^3 \leq Ra \leq 10^6$), position ($0.1 \leq x_c \leq 0.4$), length ($0.1 \leq a \leq 0.6$) and height ($0 \leq b \leq 0.4$) of the active heater on flow pattern and temperature distribution as well as heat transfer. Before turning our focus, tests of grid independence and validation of the present code are discussed.

4.1 Study of grid independence and code validation

In order to ensure grid independence of the present approach of LBM, average Nusselt number along each wall of hot centred block is computed for uniform grid size from 100*100 to 350*350 lattices number (Fig.4). From the plots, same results are produced beyond 200*200 lattices. Grid points of 240*240 is chooses for Ra=10^5 to characterize correctly the dynamic and thermal fields of the flow in enclosure. Lattices number ranging from 150*150 to 300*300 are selected for Ra < 10^5 and Ra = 10^6, respectively.
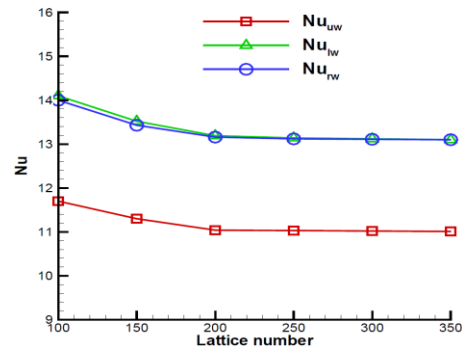

Fig.4 Average Nusselt number along each hot wall of a centred heat block versus lattice number for Ra = 10^5, a=0.1 and b=0.1

To validate the computational simulations predicted by thermal LB code, results are compared with those reported by O.Aydin on natural convection flow in square enclosure with centred discrete heat surface on the bottom wall. From the plots illustrated in Fig.5, it is found that the comparison reveals excellent agreements between streamlines and isotherms of the flow for different source length ε=0.2, ε=0.8 with fixed Rayleigh number at 10^5. Furthermore, average Nusselt numbers (Nu) along heater for Rayleigh number ranging from 10^3 to 10^6 with two source length ε are summarized in Tables 1 and 2. Numerical results show a good accuracy in comparison with same problem of O.Aydin and research of T.Naffouti on natural convection flow and heat transfer in square enclosure asymmetrically heated from below using thermal model D2Q9-D2Q9. Regarding to tables, difference of the Nu is less than 1% for different

parameters. Hence, it is concluded that these favorable comparisons corroborate the adopted approach of LB model which can produce reliable computational results.
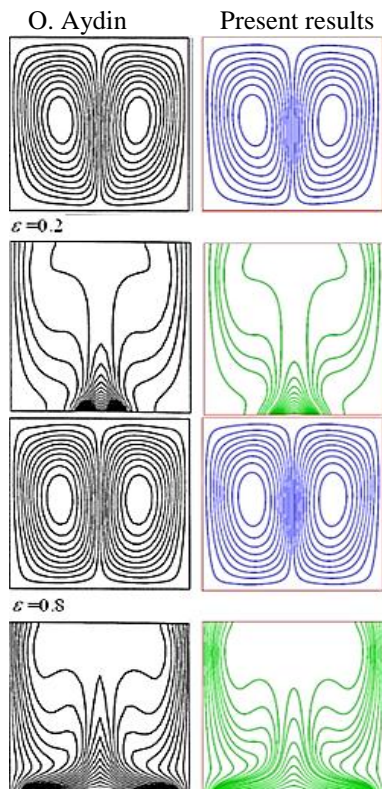

Fig.5 Comparison of the present results with O.Aydin for a centred hot source for Ra=10^5: streamlines (top) and isotherms (bottom)

Table 1 Comparison of the average Nusselt number versus Rayleigh number along hot wall for a centred hot source of length $\varepsilon = 0.2$ with O.Aydin

Ra	Present (LBM)	O. Aydin (Finite-difference)	Error (%)
10^3	1.092	1.09	0.55
10^4	1.280	1.27	0.78
10^5	2.152	2.14	0.56
10^6	3.527	3.54	0.36

Table 2 Comparison of the average Nusselt number versus Rayleigh along hot wall for a centred hot source of length $\varepsilon = 0.8$ with O.Aydin

Ra	Present (LBM)	O. Aydin (Finite-difference)	Error (%)
10^3	3.039	3.057	0.58
10^4	3.803	3.77	0.79
10^5	6.209	6.196	0.20
10^6	11.276	11.2	0.67

4.2 Dynamic and thermal structures of the fluid flow

4.2.1 Effect of Rayleigh number

Rayleigh number effect on natural convection flow in computational domain for a centred heat block with equal length and height (a=b=0.1), is illustrated in Fig.6. It is found that the flow and temperature patterns are influenced by the variation of Ra ranging from 10^3 to 10^6. For various values of Ra, it is clear that contour plots are symmetrical owing to symmetry configuration of the present problem. The streamlines are described by a pair of counter-rotating cells with clockwise and anti-clockwise rotations. In fact, thermal plume generated from heat block is blocked by upper adiabatic wall during its rise, and then it descends downwards along vertical cold walls and moves horizontally to supply again the heater in fresh air by the sides. For small Rayleigh number at 10^3, heat transfer from hot block is dissipated by conduction mode in enclosure where flow circulation is weak and temperature contours are almost vertical from cold walls. For Ra=10^4, a slight distortion of the isotherms toward cold walls of the enclosure is observed owing to an intensification of the buoyant convection flow above hot block. As increasing Ra to 10^6, buoyancy forces become more and more intense than viscous forces causing an acceleration of the flow in enclosure and a propagation of the cells cores toward upper adiabatic wall. Consequently, the entrainment intensity of heater by fresh air increases with increasing Ra. Furthermore at higher Ra, distortion of the isotherms increases in the region of the enclosure core and stratification of thermal boundary layers increases with growth of Ra indicating the enhancement of heat transfer by convection currents.
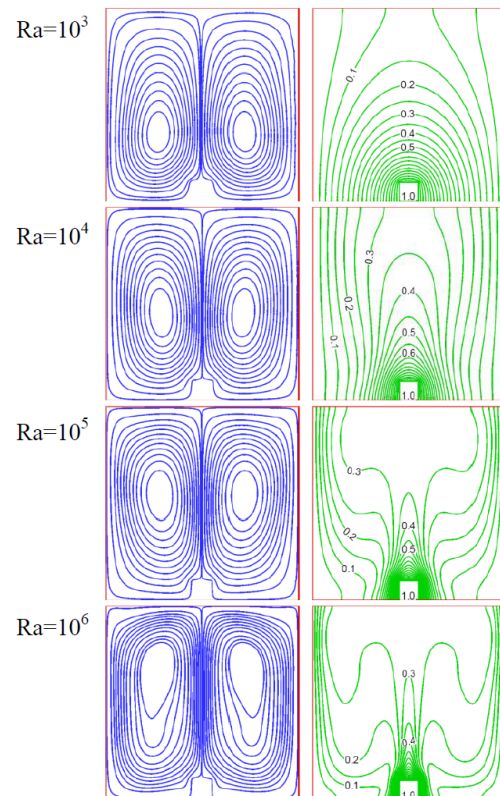

Fig.6 Streamlines (left) and isotherms (right) for a centred heat block with a=0.1 and b=0.1 at selected values of Rayleigh number

4.2.2 Effect of heat block position

Fig.7 depicts the effect of heat block position (x_c) ranging from 0.1 to 0.4, on streamlines and isotherms patterns for Ra=10^5, a=0.1 and b=0.1. It can be seen that contours plots are affected by the variation of heater position. Computational results show an asymmetric behaviour of the flow and temperature fields due to asymmetrically heating in enclosure. At x_c=0.4, the flow pattern is described by two main cells of unequal size and a slight deviation of the isotherms from vertical symmetry axis to right cold wall. Furthermore, for moving the heater toward left vertical isothermal wall, structures of dynamic and temperature fields are destroyed causing the formation of a secondary rotating cell. Owing to an intensification of flow circulation, size of the right main cell increases with decreasing source position to x_c=0.1. Consequently, the left main cell is divided in two different small cell characterized by a weak circulation of the flow in the corners of the enclosure. In addition, it is shown that thermal plume produced by hot block is more deviated to right cold wall and it descend vertically to supply again the heater. From analysis of flow behaviour, it can be conclude that the entrainment of thermal plume with fresh air increases by the right side of hot block while it decreases by the left side as source position moves from x_c=0.4 to 0.1.
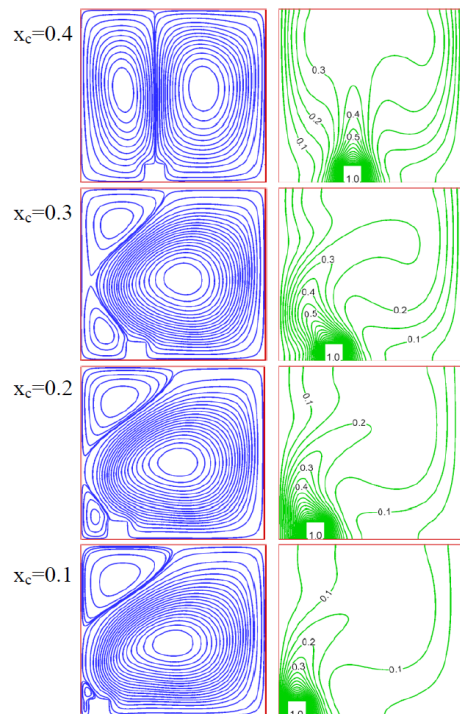

Fig.7 Streamlines (left) and isotherms (right) with a=0.1, b=0.1 and Ra = 10^5 at selected values of heat block position

4.2.3 Effect of heat block length

Fig.8 presents the effect of increasing heat block length (0.2 ≤ a ≤0.6) on flow and temperature fields in term of streamlines and isotherms, respectively for a center heater

with Ra=10^5 and heater height at 0.1. From the graphs, it can be seen that the symmetry phenomenon appears for various source length. For streamlines, the flow pattern characterized by a pair counter rotating cells (see effect of Rayleigh number on the flow in section 3.2) remains almost the same with slight deformation of contours plots on both side of heater as source length increases. However, the corresponding isotherms are affected by the variation of the length, as expected. For higher length a=0.6, contours plots of temperature become thinner near vertical cold walls of the enclosure and thermal gradients become steeper owing to intensification of convection. In addition, the degree of stratification in the vicinity of heat block increases with increasing heater length due to strong development of thermal plume generated by the active source in particularly for a=0.6. Obviously, it is related to increasing of heat transferred from hot surfaces of the block as heater length increases.
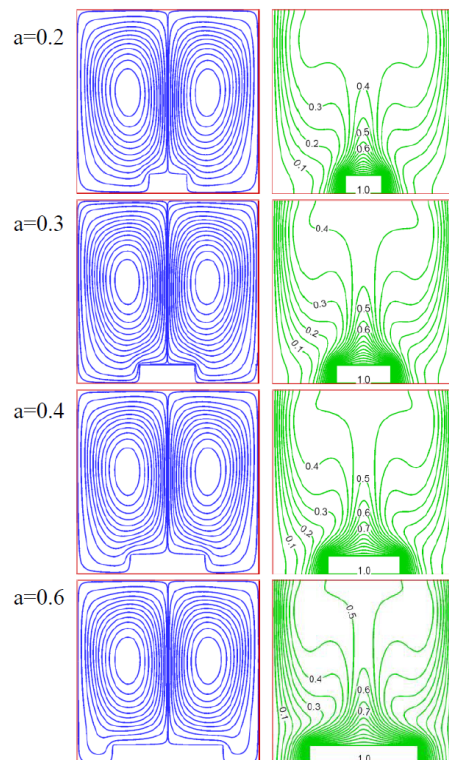

Fig.8 Streamlines (left) and isotherms (right) for a centred hot block with b=0.1 and Ra = 10^5 at selected values of heat block length

4.2.4 Effect of heat block height

Fig.9 shows the streamlines and isotherms for various values of heat block height (b) ranging from 0 to 0.4 with centred heater for Ra=10^5 and a=0.1. Analysing the graphs, it can be noticed that the flow and temperature reparation are influenced by the height of active source. For b=0 corresponding to lateral supplying of the heater with fresh air by the sides, the two symmetric convection cells formed in the domain flow are nearly identical to the problem of O.Aydin with length of discrete hot surface at 0.2. In addition, increasing the height to 0.4 leads to

deviate streamlines plots around hot block, and consequently the cores of cells move from vertical symmetry axis to isothermal boundary of the enclosure. Moreover, streamlines contours adjacent to lateral hot surfaces become more stratified as the height increases owing to intensification of vertical supplying of the heater over lateral supplying. However, the symmetric isotherms patterns become thinner and denser at thermal boundary layers and at lateral heat surfaces causing the generation of thermal gradients for various b. It is related to increase of recirculation intensity in these regions as b increases. From convection region adjacent to horizontal hot surface, thermal plume is more pushed to upward for a growth of the height especially at 0.4 by the reason of dominance of vertical supplying of the hot source.
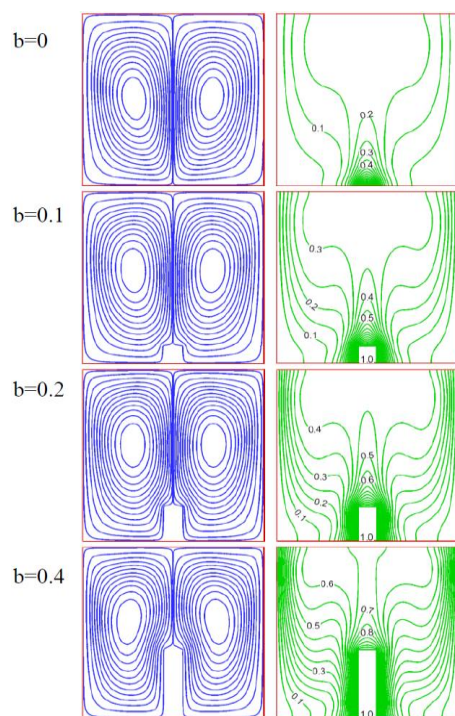

Fig.9 Streamlines (left) and isotherms (right) for a centred hot block with a=0.1 and Ra = 10^5 at selected values of heat block height (b)

4.3 Effects of different parameters on heat transfer

In order to point out the effects of Rayleigh number (Ra), position (x_c), length (a) and height (b) of the hot block on heat transfer rate, a quantitative investigation of the average Nusselt number (Nu_{uw}, Nu_{lw}, Nu_{rw} along upper, left and right walls of the heater, respectively) is revealed in Figs.10-13. By a comparison between Nu_{lw} and Nu_{rw}, it is showed that both values are the same for various parameters Ra, a and b except position x_c due to a symmetric distribution of dynamic and thermal fields.

As seen from Fig.10 for centred hot source with equal length and height (a=b), two modes of heat transfer are distinguished. For low Ra, heat transfer characterized by a weak and nearly constant Nu is mainly dominated by conduction. Beyond Ra=10^4, Nu increases rapidly with

growth of Ra than enhancing heat transfer by convection. In addition, Nu_{lw}, Nu_{rw} are more important than Nu_{uw} for high Ra owing to intensification of thermal gradient on both sides of the block. For a fixed Ra at 10^5 with equal length and height of the heater at 0.1, variation of the Nu vs. heater position $0.1 \leq x_c \leq 0.4$ is depicted in Fig.11. It can be seen that increasing x_c to 0.2 leads to decrease Nu_{uw}, Nu_{lw} about to value 11.2, therefore, the average heat transfer rate is not sensitive to source position beyond x_c=0.2. Consequently, heat transfer by convection becomes weaker as block position moves from x_c=0.1 to 0.4. In addition, it is found that Nu_{lw} is bigger than Nu_{uw} about 41% at x_c=0.1 due to strong heat transfer in the region between heater and vertical left cold wall.
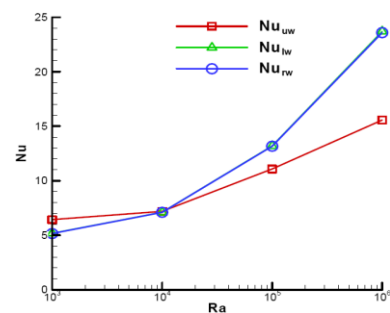

Fig. 10 Effect of Rayleigh number on the average Nusselt number along each hot wall of a centred block for a=0.1 and b=0.1
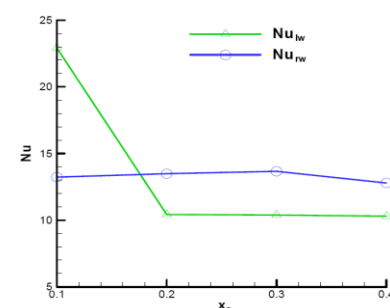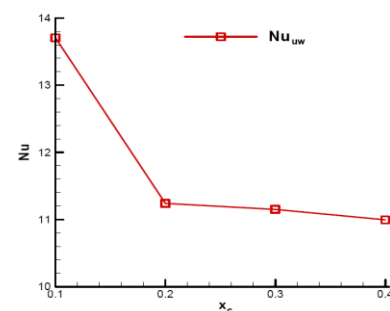

Fig.11 Effect of block position on the average Nusselt number along each hot wall of the block for a=0.1, b=0.1 and Ra = 10^5

Plots of the Nu versus length (a) and height (b) of the centred hot block at Ra=10^5 are illustrated in Figs.12 and 13. From the graphs, it can be seen that heat transfer in enclosure is very affected by the geometrical parameters a and b. For a fixed b, heat transfer by convection mode decreases with increasing of the length from 0.2 to 0.6. In

152

addition, Nu_{uw} is weaker than Nu_{lw} and Nu_{rw} due to moving of lateral hot surfaces toward isothermal boundary of the enclosure. For a=0.1, increasing the height b to 0.4 conducts to decrease Nu causing a weakening of convection heat transfer. Therefore, it is found that heat transfer rate is more significant for lower heights in particularly at b=0 corresponding to higher value of the Nu_{uw} at 15. These conclusions consolidate the view of the isothermal contours plotted in Figs.8 and 9.
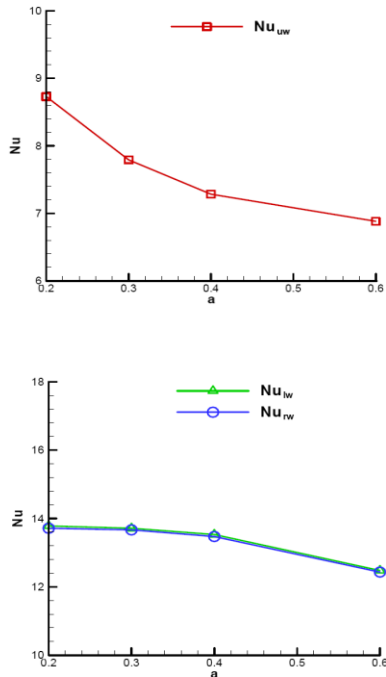

Fig.12 Effect of source length on the average Nusselt number along each hot wall of a centred block for b=0.1 and Ra = 10^5
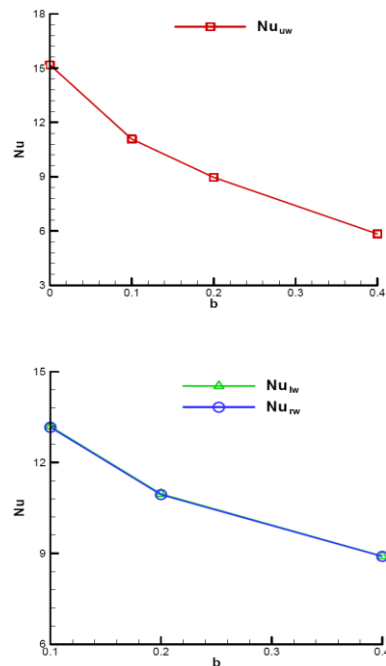

Fig.13 Effect of source height on the average Nusselt

number along each hot wall of a centred block for a=0.1 and Ra = 10^5

5. Conclusion

In this work, a lattice Boltzmann method with standard models D2Q4-D2Q9 is applied to simulate 2-D natural convection heat transfer in a square enclosure symmetrical cooled from the sides and including a rectangular heat block located on the bottom wall. This investigation enables to elucidate the effects of Rayleigh number ($10^3 \leq Ra \leq 10^6$), position ($0.1 \leq x_c \leq 0.4$), length ($0.1 \leq a \leq 0.6$) and height ($0 \leq b \leq 0.4$) of the active heater on structures of dynamic and thermal fields of the flow as well as heat transfer quantified by average Nusselt number along each hot surface. From the results, most important findings are illustrated as follows:
- The developed computational model is validated with previous study and good agreements are achieved proving the ability of this powerful LBM to simulate fluid flow and heat transfer for simple and complex problems in engineering and applied sciences.
- Graphical results in term of streamlines and isotherms are strongly affected by the pertinent parameters; Rayleigh number (Ra), position (xc), length (a) and height (b) of the active block. For various Ra, a and b the flow is described by two main cells while this behavior is completely changed and a three cell is formed as the heater moves to left vertical wall.
- Heat transfer rate along lateral surfaces of the hot block is same owing to symmetrical behavior of flow and temperature patterns in enclosure for diverse Rayleigh number, length and height except the position of active source.
- For a centred heat block with equal length and height at 0.1, convective heat transfer is enhanced for a growth of Rayleigh number especially at 10^6.
- Average Nusselt number decreases with increasing position (x_c), length (a) and height (b) of heat block. However, the higher heat transfer rate is detected for lower values of x_c, a and b with fixed Ra at 10^5.

Nomenclature

a	dimensionless length of heat block
b	dimensionless height of heat block
c_s	lattice sound speed
$\vec{\xi_i}$	discrete lattice velocity
f_i	discrete distribution function for the density
g_i	discrete distribution function for the temperature
$\vec{g}$	gravity field
h	dimensional height of heat block
l	dimensional length of heat block
L	dimensionless length of heat block
Nu_{uw}	average Nusselt number along upper hot wall
Nu_{lw}	average Nusselt number along left vertical hot wall
Nu_{rw}	average Nusselt number along right vertical hot wall
p	pressure
ΔT	temperature gradient
Δt	time step

T_h temperature of hot source
T_c temperature of cold vertical wall
Δx lattice spacing units ($=\Delta y$)
Ra   Rayleigh number
$\vec{u}$ velocity vector (u,v)
$\vec{x}$ lattice node in (x,y) coordinates
x_c position of heat block centre

Greek symbols

β Thermal expansion coefficient
θ Dimensionless temperature field
w_i weighting factors for f_i
$w^{\cdot}_i$ weighting factors for g_i
τ_f relaxation times for f_i
τ_g relaxation times for g_i
ρ fluid density
ρ_0 reference fluid density
ν kinetic viscosity
χ thermal diffusivity

Subscripts

c   cold
eq   equilibrium part
i   discrete speed directions (i=0, ..., 8)

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