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Research Article

Numerical Study of Buoyancy Induced Flow and Heat Transfer inside an Inclined Tube

Rajesh E. Thombre[A*], Shashikant B. Thombre[B] and Gajanan K. Awari[C]

[A]Rajiv Gandhi college of Engineering, Research and Technology, Chandrapur ,(M.S)., India
[B]Visveswaraya National Institute of Technology, Nagpur, (M.S), India
[C]Tulsiramji Gaikwadpatil college of Engineering and Technology, Nagpur , (M.S), India

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Abstract

The aim of the present investigation is to study numerically the heat transfer and flow characteristics for the buoyancy induced flow through inclined tubes. Mass, Momentum and energy equations were solved by finite difference approach. Constant heat flux boundary condition is created at the tube surface. Heat transfer and flow characteristics were calculated for various system parameters like tube length, tube diameter, tube inclination and heat flux supplied. The heat transfer coefficient and the induced flow rate both were found increasing with increase in the angle of inclination and heat flux supplied. The flow rate of water is found to increase with increase in the pipe length and tube diameter. The results obtained numerically were compared with the experimental data and were found within 10% agreement with each other.

Keywords: *Heat transfer coefficient, Convection, inclined tube, induced flow, Heat flux, Correlations*

1. Introduction

The flow of fluid takes place when hot body comes in contact with the cold fluid. The heat flows from hot body to the fluid due to temperature difference. The fluid expands because of heat addition, becomes lighter and moves upwards in the system. The upward movement of the fluid due to addition of heat into the system is referred as Thermo-siphon effect. Since no pump is required to cause the fluid motion, maintenance cost is lower for the thermo-siphon system and hence are preferred wherever possibility exists.

Natural convection in an inclined circular tube, has received increasing attention in recent years. This is primarily because of the numerous applications in industries. Solar water heaters, cooling of gas turbine blades, heat exchangers, cooling of electric transformers etc are a few examples to cite. The present study is carried out for the specific application, i.e. domestic solar water heating system.

The literature was reviewed for analytical and/or experimental studies on the heat transfer and flow characteristics inside a tube of different cross-sections and subjected to different boundary conditions on the heat transfer surfaces. Most of the work done was on buoyancy-induced flow through vertical or horizontal tubes, annulus and parallel plates. These configurations are never or rarely used in solar water heating system.

*Corresponding author **Rajesh E. Thombre, is** working as Associate Professor,

As far as studies on inclined tube are concerned, (Al – Arabi et At, 1987) was the first to study experimentally the natural convection heat transfer characteristics of the inclined tubes with air as the working fluid and subjected to uniform heat flux boundary condition. Similar analysis was performed by (N. Akthar, 1995) with water as the working fluid.

(Prayagi et at, 2004, 2011) has experimentally studied heat transfer and flow characteristic for buoyancy induced flow through inclined tubes and also with inserted twisted strips and varying fluid inlet temperature. He developed correlations for heat transfer coefficient and induced flow rate. But correlations developed by (Prayagi *et al*, 2004) is applicable to small range of Rayleigh number (2×10^7 to 2×10^9), for inclination ranges from 30^0 to 60^0 with horizontal, for diameter range from 12 mm to 20 mm, and for 1 m tube length to 1.5 m tube length only.

From the exhaustive literature survey, it is seen that various authors suggested different heat transfer correlations according to specific applications and specific working conditions. But numerical analysis of buoyancy induced flow through inclined tubes has not been performed by any of the investigator. Hence this work is undertaken.

In this paper, effect of tube length, tube diameter and inclination with horizontal on the heat transfer and fluid flow characteristics is studied. Mass, momentum and energy equation solved numerically and verified experimentally. Correlations for heat transfer coefficient and induced flow are suggested which is applicable to any thermo-siphon system. These generalized correlations are

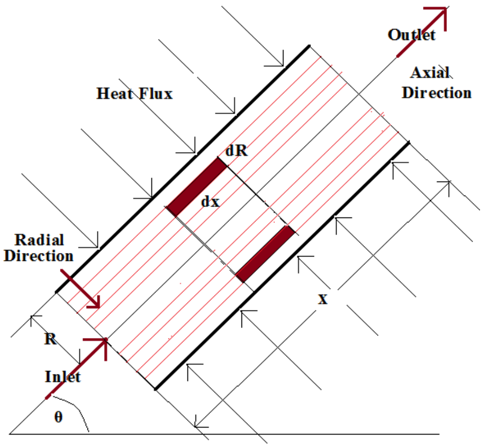

Fig 1: Diagrammatic view of inclined circular tube subjected to uniform heat flux

applicable for a wider range of Rayleigh number (1 x 10^7 to 4 x 10^{10}), for any inclinations more than 30^0, and for L/D ratio of 40 to 125.

2. Numerical Model

Figure 1 shows schematic of an inclined tube subjected to uniform heat flux boundary condition on tube surface. The fluid element gets heated up because of heat addition and its density decreases. This gives rise to buoyancy forces causing fluid element to move upward direction. It is replaced by another relatively cold fluid element from below and thus natural convective flow is established in the tube.

The fundamental equations, in cylindrical coordinates governing the flow and boundary conditions the tube is subjected to are given below

Continuity Equation

$$\frac{\partial(.\rho u)}{\partial x}+\frac{\partial(.\rho v)}{\partial r}=0 \qquad (1)$$

Momentum Equation

$$\left(u\frac{\partial u}{\partial x}\right)+\left(u\frac{\partial v}{\partial y}\right)=g\beta\sin\theta(.T-T_\infty)+\frac{\mu}{\rho}\left(\frac{1}{r}\right)\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}\right) \qquad (2)$$

Energy Equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial r^2}+\left(\frac{1}{r}\frac{\partial T}{\partial r}\right)+\left(\frac{\partial^2 T}{\partial x^2}\right)\right) \qquad (3)$$

In present paper energy equation is obtained through energy balance principle at different parts of a tube.

Boundary Conditions

Following are the boundary conditions applied for solving the equations numerically

$u=0$ At $r=R$ for all x > 0 (4)

T=Ti At x=0 for all r > 0 (5)

$$q=K\frac{\partial T}{\partial r} \quad \text{At } x>0 \text{ and } r=R \qquad (6)$$

$$\frac{\partial u}{\partial r}=0 \text{ At } r=0 \qquad (7)$$

To solve the above equations numerically, the fluid in the pipe is discretized in number of small elements, each element having independent fluid properties. Grid dependency test is conducted by varying the length ΔR and width ΔX.

Assumptions

Following assumptions were made while converting the above equation into equivalent finite difference form

1. Thermal conductivity is assumed constant.
2. Fluid properties, like density, viscosity and coefficient of volumetric expansion, are considered to be temperature dependent.
3. The Boussinque approximation is used to relate density variation with temperature.

There be n elements in X direction along the length and m elements in R directions. The numbers of nodes in X direction are n+1 and numbers of nodes in R directions are m+1. A schematic of a typical element is shown in figure 2

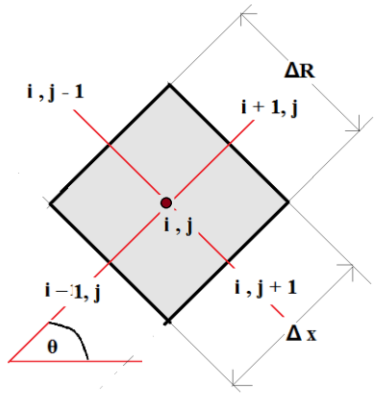

Fig 2: Fluid element i, j of length ΔR and height Δ x

The momentum equation for the element in finite difference form is written in eq.8. The energy equation is for the element in finite difference form is written in eq.9 below

$$\left(u_{i,j}\frac{u_{i,j}-u_{i-1,j}}{\Delta x}\right)=g\beta_{i,j}\,Sin\theta\left(T_{i,j}-T_{i,j-1}\right)+\left(\frac{\mu_{i,j}}{\rho_{i,j}}\right)\left(R_{i,j}\frac{u_{i,j-1}-2u_{i,j}+u_{i,j+1}}{\Delta R^2}+\frac{1}{R_{i,j}}\left(\frac{u_{i,j}-u_{i,j-1}}{\Delta R}\right)+\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{\Delta x^2}\right) \quad (8)$$

$$K\,2\,\pi.dx\left(R-\left(\frac{2j-1}{2}\right)dR\right)\frac{T_{i,j-1}-T_{i,j}}{dR}-K\,2\,\pi.dx.\left(R-\left(\frac{2j+1}{2}\right)dR\right)\frac{T_{i,j}-T_{i,j+1}}{dR}+$$
$$K\pi\left(\left(.R-\left(\frac{2j-1}{2}\right)dR\right)^{2}.-\left(.R-\left(\frac{2j+1}{2}\right)dR\right)^{2}.\right).\frac{T_{i,j}-T_{i-1,j}}{dx}.-$$
$$K\pi\left(\left(.R-\left(\frac{2j-1}{2}\right)dR\right)^{2}.-\left(R-\left(\frac{2j+1}{2}\right)dR\right)^{2}.\right)\frac{.T_{i+1,j}-T_{i,j}}{dx}+$$
$$+u_{(i-1,j)}\,\pi\left(\left(.R-\left(\frac{2j-1}{2}\right)dR\right)^{2}.-\left(.R-\left(\frac{2j+1}{2}\right)dR\right)^{2}.\right).\rho.Cp.\left(.T_{i,j}-T_{i-1,j}\right)-$$
$$u_{(i+1,j)}\,\pi.\left(\left(.R-\left(\frac{2j-1}{2}\right)dR.\right)^{2}.-\left(.R-\left(.\frac{2j+1}{2}\right).dR.\right)^{2}.\right).\rho.Cp.\left(.T_{i+1,j}-T_{i,j}\right)=0 \quad (9)$$

The fluid outlet temperature and fluid outlet velocity is evaluated by calculating average value of temperature and velocity at the outlet of the tube. Various properties of fluid are considered at average fluid temperature and are used for evaluating Reynolds number and Nusselt number.

Average temperature

$$T_{ave.\,i}=\left(\frac{\sum_{j=1,to.m}T_{i\,j}*A_{i,j}}{\sum_{j=1,to.m}A_{i,j}}\right) \quad (10)$$

Heat transfer coefficient $h_{ave\,i} = q/(T_{s\,i} - T_{ave\,i})$ (11)

Average heat transfer coefficient

$$h_{ave}=\left(\frac{\sum_{i=1.to.n}h_{ave\,i}}{n}\right) \quad (12)$$

Where n is number element along the tube length

Average velocity at the outlet of a tube is calculated as

$$u_{ave.}=\left(\frac{\sum_{j=1,to.m}u_{i\,j}*A_{i,j}}{\sum_{j=1,to.m}A_{i,j}}\right) \quad (13)$$

Average fluid outlet temperature and average heat transfer coefficients are used to calculate values of Grashoff number, Reynolds number and Nusselt number. Fluid properties like density, viscosity and volumetric coefficient of expansion are taken at fluid average outlet temperature. Various dimensionless parameters are calculated as:

Grashoff number $Gr=\left(\frac{g\beta.q.\rho.^2.D^4}{\mu^2K}\right)\mathrm{Sin}(\theta)$ (14)

Prandtl number $\mathrm{Pr}=\left(\frac{\mu.Cp}{K}\right)$ (15)

Reynolds number Re = $\left(\frac{\rho.u.D}{\mu}\right)$ (16)

Nusselt number Nu = $\left(\frac{h.D.}{K}\right)$ (17)

3. Result and Discussion

When heat is supplied to the tube, the surface temperature of the tube is increases. The density of the fluid near the surface is less than that of the fluid relatively away from the heated surface and this produces a buoyant force which causes the heated fluid to move in upward direction. Due to viscous nature of the fluid, boundary is formed near the heated wall.

The velocity next to the surface of the tube is zero due to viscous nature of the fluid. The velocity increases from zero at the wall to maximum and then decreases. This is because of buoyant force which is predominant away from the heated surface.

3.1 Velocity Distribution along the tube diameter and tube length

The figure 3 (a) and 3 (b) shows fluid velocity distribution along the radial direction at different length of a tube. Velocity of fluid is zero at surface contact of a pipe and it increases towards the center of a tube in radial direction.
For fluid elements near to the inlet of the tube velocity of fluid increases near the surface of a tube and it reduces towards the center and become constant near the center i.e

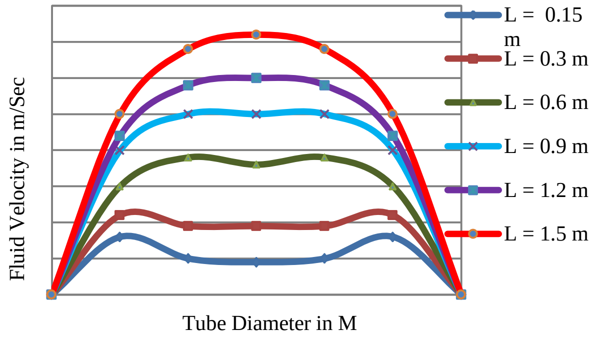

Fig 3 (a): Velocity distributions along the tube diameter

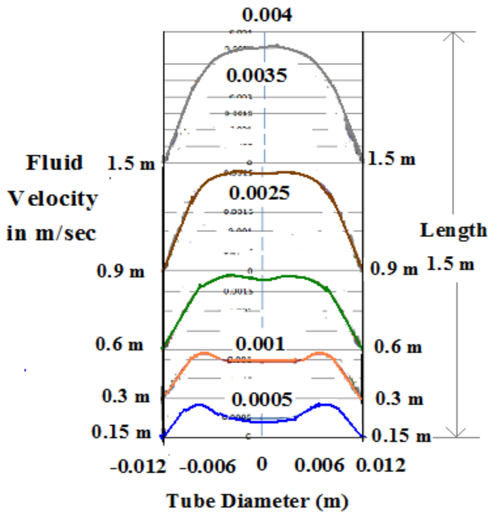

Fig 3 (b): Velocity distributions along the tube diameter and length

a bowl shape velocity profile is observed in 3D geometry (when viewed in 2D, two peaks are observed). In this region the hydrodynamic boundary is developing. As flow advances in the pipe, the two peaks come closer from all along the circumference of the tube. Ultimately merges with each other and a single parabolic profile, (maximum value occurring at the center) is obtained indicating thereby that boundary layer is now fully developed. This situation is found to occur at L/D ratio of around 66. Due to continuous addition of heat along the length the fluid flow velocity increases with the tube length also heat flux supplied. The vertical tube configuration yielded higher velocity compared to the inclined tube configuration. This is because the component of the buoyancy force along the tube length is significantly larger in case of vertical tube and grows smaller and smaller as the tube inclination with the horizontal decreases.

3.2 Temperature distribution along the tube diameter and tube length

Temperature variation along the tube radius is evaluated numerically and is shown in figure 4. It is observed that temperature of fluid increases along the length of a tube. Temperature at the surface is increases. Temperature of fluid is highest at the surface and it reduces towards the center of the tube parabolically. This variation is as per expectations.

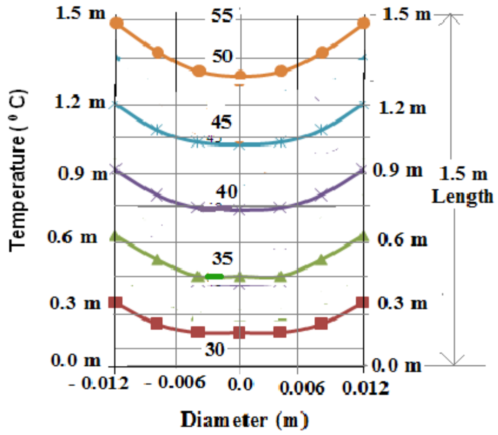

Fig 4: Temperature distributions inside a tube along the diameter and length

Figure 4 shows that, near the entry to the tube, the variation of temperature is more or less flat as compared to temperature distribution near the exit of a tube, where it is a bit deep and cup type. As tube length increases the average fluid outlet temperature increases. Similar variations are observed for different tube diameters but the average fluid temperature at the exit is found to be a bit less for larger diameter for same heat addition. This is obvious because larger diameter tube holds more water compared to the smaller diameter tube for the same tube length. This rise in temperature of water for same heat addition is found to be less in case of the vertical configuration compared to inclined configuration. This is also as per expectation because for the reasons mentioned earlier, the flow rate is maximum for the vertical configuration and decreases for the inclined configuration.

3.3 Local Temperature Variation

Local temperature distribution along the tube length obtained numerically is shown in figure 5. Temperatures at various locations on the tube surface are calculated by solving the numerical equations no (9) and (10). Average fluid temperature is calculated by equation (11).

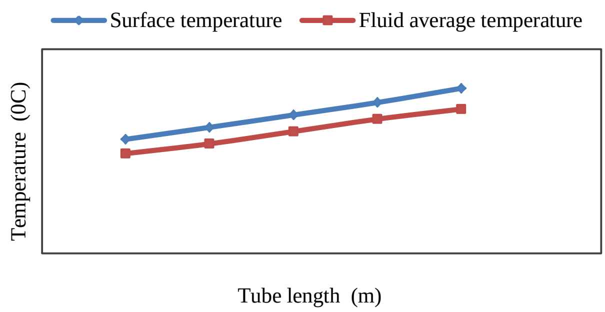

Fig 5: Local temperature distribution along the length of tube obtained by numerical equations.

It is observed that the surface temperature and fluid average temperature increases along the length of the tube. The difference between surface temperature and fluid average temperature remain approximately constant along the length of tube. This is a typical variation for fully development flow inside the tube subjected to uniform heat flux boundary condition. The temperature difference between the tube surface and fluid average temperature is found to decrease with tube increase in tube inclination with respect to horizontal, for same heat flux leading to higher values of heat transfer coefficient, This is discussed in next section.

Similar results were obtained experimentally as well. The difference between surface temperature and fluid temperature becomes constant after length of 0.75 m. Hence the flow becomes fully developed after the tube length of 0.8 m.

3.4 Local heat transfer coefficient

Figure 6 shows the variation of local heat transfer coefficient estimated numerically and experimentally along the tube length. The local heat transfer coefficient is calculated by equation (12). As seen from the figure 5, the local heat transfer coefficients calculated numerically, initially decreases and attain a constant or fully developed value at about 0.5 m tube length from leading edge. The fully developed value for 24 mm tube diameter and 1.5 m length in vertical position is 310 W/m^{2}K. Experimentally determined local heat transfer coefficient also shows similar variation but the value is obtained at about 0.75 m from the leading edge and the fully developed value is 297 W/m^{2}K. Slight increase in heat transfer coefficient near the outlet of a tube, is due to reduction in temperature gradient. This may be because of more heat loss to the atmosphere, as extended end of a tube is not insulated. This loss is not accounted for calculations. The numerically predicted and experimentally obtained values of heat transfer are within ± 4.5% of each other.

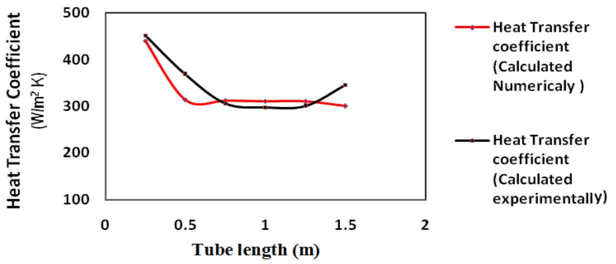

Fig 6: Variation of local heat transfer coefficient along the tube length

Similar results were obtained for small tubes but with different numerical values. It is found that for same heat flux supplied, the heat transfer coefficient reduces with reduction in tube diameter and increase in tube length. This is due to fact that for small diameter and small length of tube, more heat is collected by the fluid flows through small area. Also flow become fully developed at small length and hence less heat is collected by the fluid for large tube length.

3.5Heat transfer coefficient with respect to tube inclination

Figure 7 shows the effect of tube inclination on heat transfer coefficient for different tube diameters. It is observed from the figure that the heat transfer coefficient increases with increase in tube inclination. This is because, as stated earlier, the buoyancy induced flow in the tube increases with the tube inclination and thus washes the tube surface with greater velocity. Typically for every increase in inclination, the heat transfer coefficient increases by 10 to 15 %.

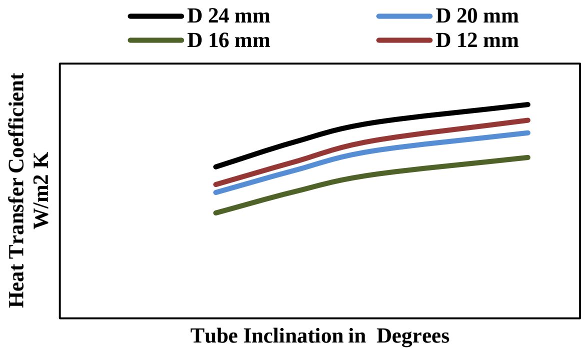

Fig 7: Effect of tube inclination on heat transfer coefficient

3.6 Induced Flow Rate With Respect to Tube Length and Tube Diameter

Figure 8 show the change in velocity of fluid with respect to L/D ratio. It is observed that the exit fluid velocity increases with increase in L/D ratio for all inclinations. Fluid velocity increases with increase in length for same diameter of a tube. It is observed that by doubling the L/D ratio, fluid velocity increases by about 25-30%. This is obvious because pipe is subjected to uniform heat flux boundary condition and with increase in length, the heat that is pumped into the fluid inside the pipe also increases, causing thereby rise in temperature of the fluid and thus increasing the buoyancy force.

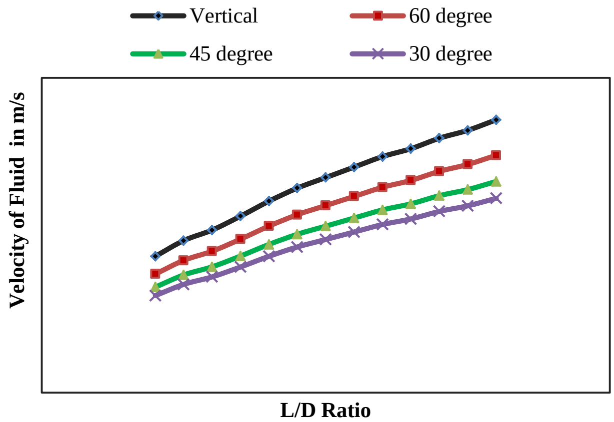

Fig 8: Velocity of fluid with respect to L/D ratio

Figure 8 also show that the flow rate is highest in vertical position and it reduced with reduction in tube inclination. The reason has already been discussed. The flow rate is reduced by about 12% with reduction in inclination from vertical to 60 degrees and reduced by about 6 to 7% with reducing inclination from 45 degrees to 30 degrees.

3.7 Flow characteristics Results

The variation between Re, Gr and L/D ratio is given by

$$Re = C_1 (Gr\ Pr\ L/D)^n \qquad (19)$$

The Gr and Re are calculated by equation no (15) and (17). The tube inclination is taken care by the definition of Gr; length and diameter are taken care by L/D ratio. Figure 9 shows the characteristics (Re Vs. Gr) for buoyancy induced flow inside an inclined tube. It can be seen from the figure that Re (induced flow) increases with the increase in Gr (heat absorbed) and represents L/D ratio. Typically it is observed that by 50% increase in Grashoff number, Reynolds number is increases by about 29%, similarly for 33% change in L/D ratio, Re increases by about 16%. With increase in angle from 30 degrees to 45 degrees, Reynolds number increases by about 10% and from 45 degrees to 60 degrees Re increases by about 15%.

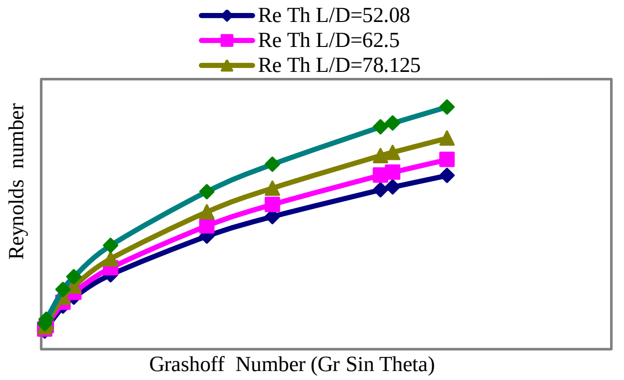

Fig 9 Change in Reynolds Number with Grashoff number.

The value of constant C_1 and index n are calculated by generating numerical data for different tube configurations and by regression analysis are calculated as 0.004 and 0.5 respectively. The equation (19) thus becomes

$$\text{Reynolds number } Re = 0.004\ (Gr.Pr\ \sin\theta.\ L/D)^{0.5} \qquad (20)$$

The fluid properties are evaluated at mean fluid outlet temperature. The above equation (20) is applicable over the range

$40 < L/D < 125$
$5x10^5 < Gr < 5 \times 10^8$
$2.0 < Pr < 7.0$
$30^0 < \theta < 90^0$
$2 \times 10^7 < Ra < 4 \times 10^{10}$

3.8. Heat Transfer Characteristics

Figure 10 shows heat transfer characteristics (Nu vs. Ra) for buoyancy induced flow inside an inclined tube. Nusselt number evaluated using fully developed heat transfer coefficient as described in section 3.4. As expected, it is observed that Nusselt number increases with increase in Rayleigh number. It is seen that for vertical tube configuration, 50% increase in Rayleigh Number, the Nusselt number is increase by about 30 %. Reduction in Nusselt number is observed with reducing the inclination of tube with respect to horizontal. With decrease in inclination from 90^0 to 30^0, Nusselt number is by about 25%.

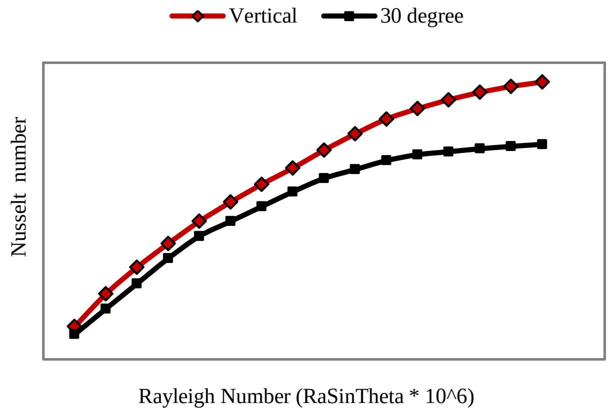

Fig 10: Nusselt Number Vs. Rayleigh Number

The relationship between Nu, Ra and θ is given by

$$\text{Nusselt Number } \mathbf{Nu = 0.0036\ (Ra\ .Sin\ \theta\ )^{0.5}} \qquad (21)$$

The equation is applicable over the range

$40 < L/D < 125$
$5x10^5 < Gr < 5 \times 10^8$
$2.0 < Pr < 7.0$
$30^0 < \theta < 90^0$
$2 \times 10^7 < Ra < 4 \times 10^{10}$

Results obtained by numerical analysis are compared with the experimental results. Figure 11 shows graph between Nusselt number obtained numerically and experimentally for all the configurations investigated. It can be seen from the figure that the results obtained by numerical analysis and those obtained experimentally are in good agreement with each other and are 10% of the mean.

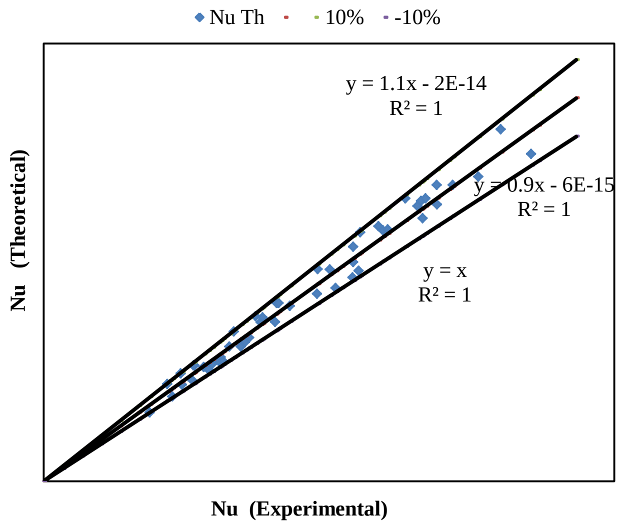

Fig 1: Comparison of numerical results with experimental results

Conclusions

Numerical analysis for buoyancy induced flow inside an inclined pipe carried out for different tube dimensions, tube inclinations and heat flux supplied. The following observations are made.

- Fluid velocity increases near the surface of a tube and it reduces towards the center and become constant near the center i.e a bowl shape velocity profile is observed in 3D geometry (when viewed in 2D, two peaks are observed). This indicates that the hydrodynamic boundary is developing in the region.
- As flow advances in the tube, the two peaks come closer from all along the circumference of the tube and ultimately merge with each other and a single parabolic profile is obtained indicating that boundary layer is fully developed. This is found to occur at L/D ratio of around 42 i.e. around length of 0.8 m from the entry of the tube.
- The average heat transfer coefficient and the average induced flow rate both increase with increase in the angle of inclination of the tube with the horizontal.
- The average heat transfer coefficient increases with increase in tube diameter.
- Reynolds number increases with increase in tube inclination but it reduces with increase in tube diameter. The increase in Reynolds number is due to increase in buoyancy force, which results in higher mass flow rate.
- Results obtained numerically and experimentally are within of 10% of the mean.
- The heat transfer coefficient and induced flow rate are strongly influenced by heat flux, diameter of the tube, tube length and its inclination. Balance between these factors is essential to enhance the flow rate and heat transfer coefficient.

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